

Hall probe, Magnetic Field and Forces

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Introduction

In this lab you will perform a series of experiments that will familiarize you with magnetic fields, apparatus used to measure magnetic field (the Hall probe) and magnetic forces and interactions. You will examine the behavior of a magnetic dipole in both uniform and non-uniform magnetic fields and will take data that will allow you to calculate the magnetic dipole moment of your magnet. In the end of the lab with the help of your instructor you will observe the precession and “spin flip” of a rotating magnetic dipole.

List of tasks to be performed.

A detailed description of each task is given in the **Experiments** section below.

1. Calibrate the Hall Probe in a (almost) Helmholtz coil.
2. Measure the variation of the field of an (almost) Helmholtz coil.
3. Measure the gradient of an opposing Helmholtz coil.
4. Determine magnetic dipole moment of a magnet in a snooker ball.
5. Measure the force on a dipole magnet in uniform and in gradient magnetic field.
6. Observe the precession of a spinning ball with a magnetic moment in a field.
7. Observe the flip of a precessing, spinning ball a with magnetic moment.

Apparatus

The apparatus (from TeachSpin) consists of several components described below.

1. The Hall probe.

A Hall probe is an instrument which is used to measure magnetic fields, based on the Hall effect. It is a small, rectangular sheet of semiconductor which measures the component of the magnetic field which is perpendicular to the sheet. A current is sent through the sheet from one edge to the opposite edge. The $v \times B$ force on the conduction electrons produces a charge accumulation on the other two edges of the sheet. At equilibrium, the electric field of this charge accumulation balances the $v \times B$ force. This electric field times the width of the sheet of semiconductor yields a voltage across the probe which can be read on a high impedance voltmeter. Figure 1 shows a schematic view of the forces and fields in the Hall probe. Figure 2 shows a picture of the probe used in this experiment.

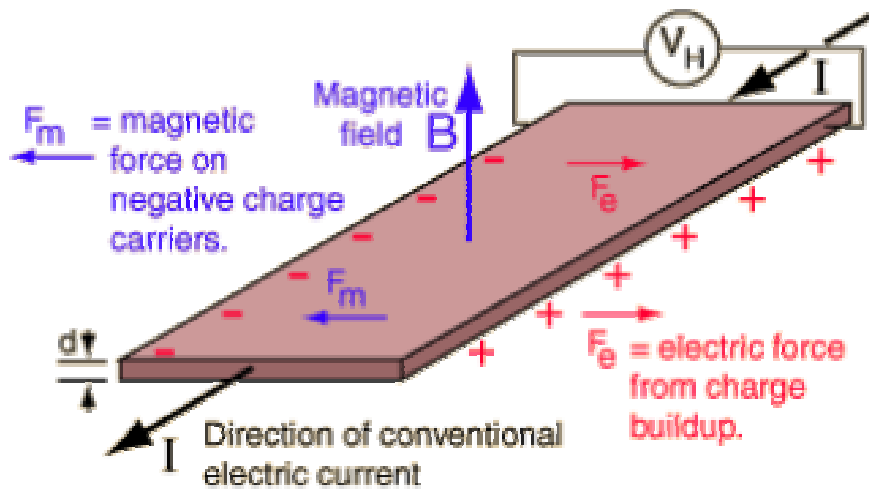


Figure 1. Schematic view of the forces and fields in a Hall probe. The figure is taken from the Georgia State University HyperPhysics website¹ (excellent resource to look-up introductory physics concepts).

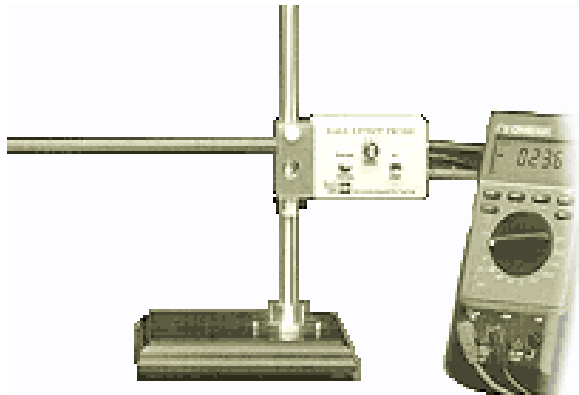


Figure 2. The Hall probe used in this experiment. (Figure from TeachSpin)².

2. The Magnet

The most obvious way to make a magnetic field is with a long solenoid. The resulting magnetic field is parallel to the axis of the coil and very uniform throughout the volume except near the ends. The disadvantages of using a solenoid are: 1) The power consumption and heat developed become problems even for rather low fields. 2) It is hard to bring instrumentation in from the side because the coil is in the way. A compromise between field intensity and uniformity is to use a pair of coils, equivalent to leaving out a region of turns in the solenoid (and saving the power which would have been dissipated in them). If the two coils are separated by a distance equal to the radius, the field near the center is particularly uniform and this combination is called a Helmholtz coil. Some increase in field and a slight degradation of uniformity results from placing the coils slightly closer than the Helmholtz condition. The coils that you will use have been optimized for a slightly higher field. They are composed of copper wire that is wound on bobbins. Each coil has 195 turns. The two coils are always connected in series so that the same current flows through each turn. If the current in one of the coils is reversed, the field at the midpoint of

¹ <http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>

² <http://www.teachspin.com/>

the coils must vanish by symmetry. It is easy to show³ that, for points on the axis and near the center, the field is axial and the magnitude of the field increases linearly with distance from the center.



Figure 3. The Magnet and power supply from TeachSpin.

3. Power supply

The original apparatus came with a power supply, shown next to the Magnet in Figure 3. However, this power supply was damaged (by switching large currents on and off using the switch, instead of bringing the current down gradually). Presently, we are using an external power supply. The analog ammeter from the original power supply still works and you can use it to measure the current through the coils. It is important to note that the coils have some resistance, and that resistance is temperature-dependent. If current is allowed to flow through the coils for a long time, or if the current is high (~3-4 amps), the coils' temperature begins to rise. You can feel the increase in temperature and take measures to turn the current down to zero when the magnet is not being used.

4. Air bearing

The air bearing is the spherical hollow in the cylindrical brass rod that is supported on the bottom coil form. The bearing has a narrow opening that allows air to be pumped into the spherical hollow. The ball sits in this hollow and floats on a cushion of air. This provides support without significant friction. The air pump is housed inside of the power supply. A vinyl hose attaches to the back of the power supply at one of its ends, and at the other end attaches by a threaded right-angle fitting to the under-side of the air bearing. Make sure not to restrict air flow by accidentally "kinking" the hose.

5. Strobe light

The strobe light is located in an insulated housing on top of the upper coil. One can vary the frequency of the strobe flashes by a control located on the power supply front panel. The frequency of these flashes is automatically measured and read out to two significant figures on the power supply's front panel. This data is updated every 10 seconds.

6. Strobe light frequency display

Displays the frequency of the strobe light in Hertz. Below the display is the frequency adjust. Turning the knob clockwise will increase the frequency. After adjusting the frequency, one needs to wait for the instrument to count up to the actual frequency. Next to the frequency-adjust knob is the on-off switch for the strobe light.

7. Air switch

Allows you to turn off the air pump when you are performing the magnetic force experiments.

³ Any introductory physics text should give you enough knowledge to be able to calculate the magnetic field along the axis near the center of the apparatus, using Biot-Savart's Law and the principle of superposition. If you wish to try this, you can use the following numbers:

equivalent radius of the coils = 0.109 m

equivalent separation between the coils = 0.138 m

8. Pilot light

Indicates when the ac power is on for the entire system inside the power supply case.

9. Accessories

These are components that used in various experiment and are not permanently attached to the two main parts of the instrument.

a. Cue balls

These cue balls (see pictures) are simply aramith snooker balls with a small cylindrical permanent magnet at their centers that acts as if it were a magnetic dipole. The magnetic dipole moment points in the direction of the ball's handle. The handle allows you to spin the ball, measure its rotation frequency, and determine the direction of the magnetic moment. The handles on the balls have a small axial hole drilled in them. In this hole, a thin metal rod with an attached weight is placed as part of a static magnetic torque experiment. The aluminum rod has a steel tip at one end that holds fast to the magnet inside of the ball. The movable weight is a small clear plastic cylinder with an O-ring inside that keeps the weight from involuntarily slipping on the rod. The weight is meant to be moved up and down the rod to vary the gravitational torque (Figure 4).

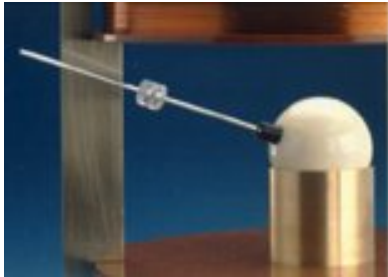


Figure 4. Cue ball with weight attached as used in the magnetic vs gravitational torque experiment.

b. Plastic tower

The clear plastic tube attached to a cylindrical base can be placed on top of the air bearing (Figure 5 (left)). This apparatus is used for the magnetic force experiment. A nylon cap placed on the top of the tube that holds the rod that supports the spring. The other end of the spring is connected to the magnet. The position of the suspended magnet inside or the tube can be adjusted by moving the rod inside the cap. There is also a small screw-eye attached above the magnetic dipole that can be used to prevent the magnet from rotating on its gimbals. Ball bearings that weigh one gram each are provided for calibration of the spring constant.

c. Rotating magnetic field

A special configuration of permanent magnets and soft iron shims is provided to generate a horizontal magnetic field. This uniform horizontal magnetic field (~ 1.0 mT) can be manually rotated around the air bearing. It has a hole in its base that allows the air bearing to act as its rotation axis (Figure 5 (right)). This magnetic field is used to demonstrate nuclear resonance.

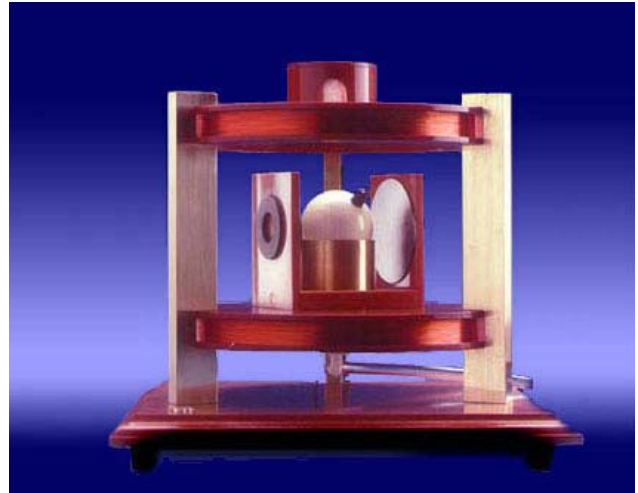


Figure 5. (left) Plastic tower with spring and magnet. (right) Rotating (horizontal) magnetic field.

10. Additional notes on apparatus:

Field direction switch: Controls whether the magnetic field at the center is either up or down.

Field gradient switch: Controls whether there is a magnetic field gradient at the center of the coils, or a uniform magnetic field (gradient off).

On the back of the power supply are the following:

1. --on-off ac switch for all of the components inside.
2. --cord that plugs into the ac electrical socket.
3. --Cinch Jones connector that connects the power supply to the magnet.
4. --male air hose connection -- the air hose has a female connection that mates to it.

Experiments:

1. Calibrate the Hall Probe in a (almost) Helmholtz coil.

The Hall probe is used to measure magnetic fields. But before you can use any measuring device, you need to calibrate it. You will calibrate the Hall probe by first measuring the Hall voltage for *known* magnetic fields. The Hall voltage (V_H) is accurately proportional to the magnetic field, but the coefficient of proportionality depends upon the drift velocity of conduction electrons in the probe. Since some geometric factors and the free electron density are not well known, the proportionality must be measured as a calibration constant. In addition, it is common that you will use an amplifier to measure the Hall voltage. The gain of the amplifier is another factor that is accounted for with the calibration. In order to calibrate your device, you need a way to measure the magnetic field independently from the probe measurement. We will do so by measuring the current through the coils. For the coils in this experiment, $B = 1.36 \times 10^{-3}I$, with I in amperes and B in tesla.

There are two probes in the tube and the manufacturer's write up lists them as located at a centimeter from the end of the tube. The planes of the two probes are perpendicular. One connection is labeled "radial" and the other "trans". If the probe is mounted on the stand with the writing on the side, the "trans" position of the switch connects to the Hall probe which measures a vertical field. Use the "x10" setting. Put the Hall probe at the center of the pair of coils. Set the zero carefully with no current in the Helmholtz coils, and then record the voltmeter readings for settings of 1, 2, 3, 4, and 4.5 amperes in the coils. Tabulate your values for I , B and V_H . Plot B vs V_H . Is the relation between B and V_H linear? Fit the graph with a straight line and obtain the calibration constant.

All of the work for this experiment can be done with the "x10" sensitivity, but it will help you develop an appropriate cynicism for believing all you read to check the ratio of readings with the "x1" and "x10" label on the sensitivity range switch for a couple of coil currents.

2. Determine the region of uniform field

The experiments that you will perform will use the field in the center of the coil (which should be nearly uniform). In this part of the lab you should verify that using the Hall probe. Pick one value of the current in the coils and measure the magnetic field along the axis of the coils and along a radius in the mid plane. Extend your measurements from the mid point to the limit of motion of the probe. For the vertical transit, it may be helpful to place the probe so that it is next to one of the vertical supports of the coil to provide a mechanical guide. For the horizontal transit, simply rotate the probe on the stand and measure the displacement from the line through the hole in the bottom of the brass cup. Tabulate your data for V_H , B , z (vertical coordinate), or r (radial distance) and make graphs of B vs r and B vs z .

3. Measure the gradient of the magnetic field in opposing coils

When the currents in the two coils are in opposite directions, the field at the mid point is zero by symmetry. Near the center of the coils, the vertical component of the magnetic field is proportional to the distance above the center; i.e. – there is a uniform field gradient near the center. Be sure that the power supply is off when you make the switch and turn the gradient switch on. With the current at three or four amperes, make a set of measurements starting at a probe location close to the brass cup and continuing up to the position at which the probe hits the top coil. Plot your data with 0 on the z axis at the height measured for your best estimate of the mid point between the coils. There may be some deviation from linearity as you go away from the center. Fit the graph B vs z with a linear function. Does the fitted intercept agree with your best estimate? The slope of the line is the gradient dB/dz . Note at what current through the coils you are making the measurement. Since the gradient in the field is proportional to the current, having done this measurement for one value of I , you should be able to determine dB/dz for other values of I . You will need this in Experiment xxx .

4. Determine the magnetic dipole moment of a magnet in a snooker ball. Magnetic torque equals gravitational torque.

The main objective of this experiment is to measure the magnetic moment (μ) of a magnetic dipole (which is the magnet inside the cue ball). You will do this by balancing the magnetic and the gravitational torque. You will use the Magnet, air bearing, cue ball, aluminum rod, weight, ruler, and calipers. The set-up for this experiment is shown in Figure 4.

In a uniform magnetic field (which is the case at the center of the two-coil configuration with the gradient switch in "off" position), a magnetic dipole experiences a magnetic torque, τ , which is given by the expression:

$$\tau = \mu \times B,$$

where μ is the magnetic dipole moment. Your magnet's dipole moment is aligned parallel to the handle on the ball; the magnetic field produced by the coils can be either up or down along the coils' axis. If the magnetic field points up, and the magnetic moment is aligned at some angle θ away from the direction of the magnetic field, the ball will experience a torque that will tend to rotate it so that the handle of the ball points upward. But if the aluminum rod is placed in the handle of the ball, there is now another torque due to the earth's gravitational field. The expression for this torque is:

$$\tau = r \times mg$$

The gravitational torque tends to cause the ball to rotate so that the ball's handle points downward. Since a net torque causes change in angular momentum, the ball will rotate if the gravitational torque is larger than the magnetic torque, or vice versa. But when the magnetic torque is equal to the gravitational torque, the ball will not rotate, since the net torque on the ball is zero. This configuration is mathematically represented by:

$$\mu B \sin\theta = rmg \sin\theta$$

Or

$$\mu B = rmg \quad r = \frac{\mu}{mg} B$$

We can slide the weight on the aluminum rod (i.e. - vary r), and measure the magnetic fields that are needed such that the magnetic torque will balance the gravitational torque. The functional dependence of r to B should be a straight line with the slope being an expression that contains μ . From the equation above it is easy to determine B by measuring the current that it takes to get the ball in equilibrium. Determining r and m requires more thought. In principle, there are two m 's and two r 's that we need to consider. The first m is the mass of the weight, with its corresponding r being the distance from the center of the ball to the center of mass of the weight. The second m is the mass of the rod, with its corresponding r being the distance from the center of the ball to the center-of-mass of the rod. One way to take this into account is to measure the magnetic field needed to balance the rod without the weight and then subtract this field from every subsequent measurement. Another way (less labor consuming) is to remember that we can measure μ using the slope of a line. It turns out that the mass of the rod and its center-of-mass distance, r , are combined in a constant in the graph of r vs. B , and do not affect the slope of the line. So the only r that one needs to measure is the r of the weight, and the only m that must be measured is the mass of the weight. This is the advantage of a slope measurement of μ rather than a single point determination where the mass and center-of-mass of the rod would be essential. The following numbers were measured by your instructor: the mass of the weight is $m = 1.4 \pm 0.1$ g and the diameter of the cue ball (measured with a caliper) is 53.85 ± 0.01 mm.

Procedure:

Select a position for the weight. Measure r for this position. Increase the current until you balance the two torques. You will typically find that you will need currents > 2.5 Amps. The equilibrium is unstable – you will need to use the fine adjustment to get the ball stable. However, note by how much the current changes if you tweak the knob slightly and you lose the equilibrium. What is the error in your measurements of the current and the magnetic field at equilibrium? What is your error in r ? Determine the % error in μ , assuming that you determine it from a single point measurement using the formula above. Since you are using the slope of a line, your measurement will have slightly smaller statistical error. It will also be free of systematic errors that affect the offset, but not the slope of the line. Tabulate your data and put the graph with the fitted line in the report. Determine the dipole moment μ and its error.

5. Measure the monopole and dipole moment of a sample magnet.

For this experiment you will use the plastic tower and a magnet suspended on a spring (see Figure 5) to measure the magnetic monopole and the magnetic dipole moment of a sample magnet. Although most introductory E&M books do not discuss magnetic monopoles, there is no first principle reason on why magnetic monopoles should not exist. If they do, we could measure the magnetic monopole moment in the same manner as we would do for the electric monopole moment (usually called charge), i.e. by measuring the force on the monopole in an external field. The dipole moment, from the other hand, will experience a force in a gradient field. In both cases we would need to measure the force on the magnet, so we will first set-up and calibrate a device which will allow us to measure forces. That is why we use the spring. You remember from mechanics that the elastic force of the spring is given by Hooke's law:

$$F = kz,$$

where k is the spring constant and z is the elongation of the spring. If we know k , we can determine the force by measuring the elongation. To determine k we are given several balls with weight of 1 g each. You can increase the mass suspended by the spring in steps of 1 gram and measure the corresponding elongation. In equilibrium:

$$F = kz = mg$$

Again, using a slope of a graph maybe the best way to go. Tabulate your data of z and m . Make a graph, determine k . Estimate the % error in k assuming that you know m and g exactly. Once you know how to

measure forces, you are ready to measure the monopole and the dipole moment of your magnet.

Monopole moment: The monopole moment is given by the ratio of force to field in a uniform external field. Take several measurements of the displacement of the magnet, z , at different magnetic fields (keep the gradient of the field off). Determine the magnetic force for each B . Plot F vs B and fit it with a line. Is the slope different from 0? Interpret your result.

Dipole moment: The dipole moment is the ratio of force to field gradient (dB/dz). For several different current settings with the gradient of the field "on", measure the displacement of the magnet, z , and determine the magnetic force. Use your measurement of dB/dz from experiment 3 and scale appropriately with the current to get the values at each current setting. Plot your data and determine the dipole moment from the slope of the graph F vs dB/dz .

- 6. Observe the precession of a spinning ball with a magnetic moment in an external field.**
- 7. Observe the flip of a precessing, spinning ball with magnetic moment.**

Experiments 6 and 7 will be done with your instructor. Discuss with her the physics in the observed phenomena and write a short (couple of sentences) summary of what you observed in each of these experiments.