Lecture 01

- Experimental facilities for relativistic heavy ion (RHI) research
- Natural units in RHI physics
- Relativistic kinematics
  - 4 vectors
  - Lorentz transformations
  - Some convenient variables:
    - Transverse mass
    - Rapidity
    - Pseudo-rapidity
  - Lorentz invariants

Heavy Ion Physics

- Low energy (a few MeV/nucleon)
  - Accessible in electrostatic tandem Van de Graaf accelerators followed by a LINAC (uses RF frequency AC field)
  - Two nuclei collide: if energy is enough to overcome the Coulomb barrier – compound nucleus is formed, which is left in an excited state
  - The excited nucleus may evaporate particles (p,n α) or it can fission
  - The nucleus maybe also be broken-up in lighter nuclear fragments
  - To produce new particles: need center of mass energy higher than the mass of the particle
    - m_π ~140 MeV, m_k ~ 500 MeV
- Relativistic heavy ion collisions
  - As the energy increases more and more various particle production thresholds are exceeded.
  - With high enough temperature, the relevant degrees of freedom become quarks and gluons

The Relativistic Heavy Ion Collider

RHIC 2000 Run
Au+Au
$\sqrt{s_{nn}}$ = 130 GeV

RHIC 2001-2 Au+Au & p+p Run
$\sqrt{s_{nn}}$ = 200 GeV

RHIC 2003 d+Au & p+p Run
$\sqrt{s_{nn}}$ = 200 GeV

RHIC 2004 Au+Au & p+p Run
$\sqrt{s_{nn}}$ = 200 GeV, 62 GeV

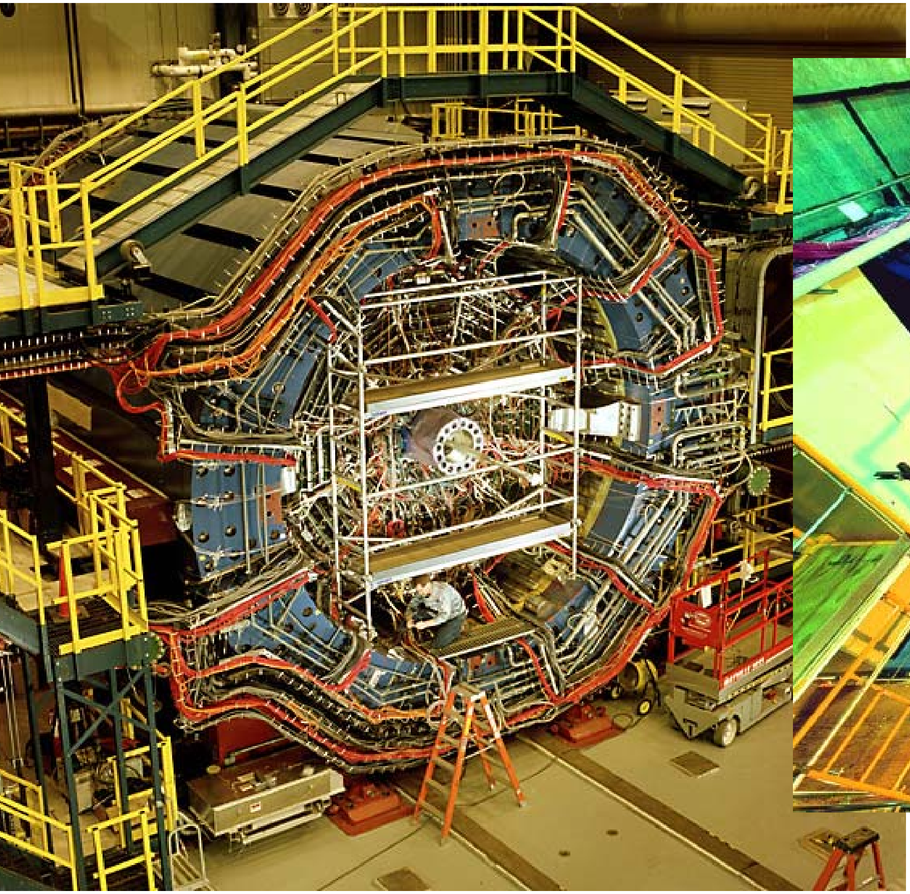
RHIC 2005 Cu+Cu Run
$\sqrt{s_{nn}}$ = 200 GeV, 62 GeV, 22 GeV

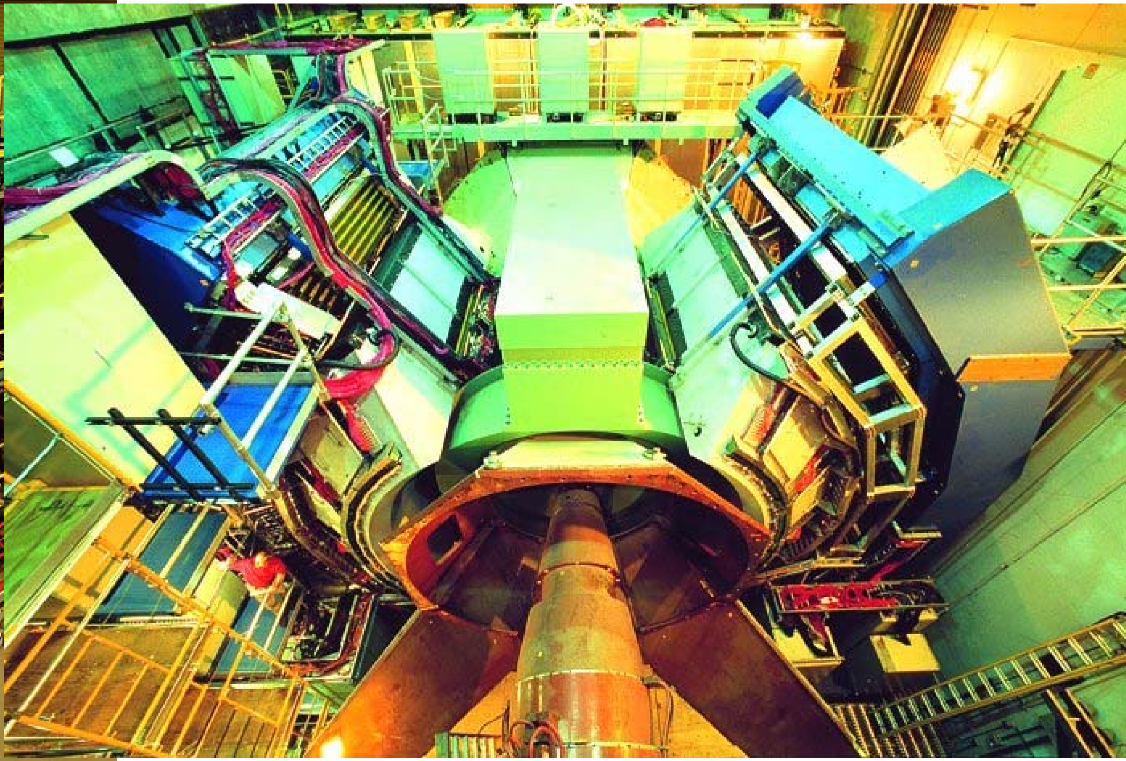
RHIC 2006 p+p Run
$\sqrt{s_{nn}}$ = 200 GeV

RHIC 2007 Au+Au Run
$\sqrt{s_{nn}}$ = 200 GeV

- **2 counter-circulating rings**
  - **2.4 miles circumference**
  - **1740 super conducting magnets**
- **Collides any nucleus on any other**
- **Top energies: 200 GeV Au-Au**
  500 GeV polarized p-p
- **Four experiments: BRAHMS, PHOBOS**
  PHENIX, STAR

The Big Experiments at RHIC

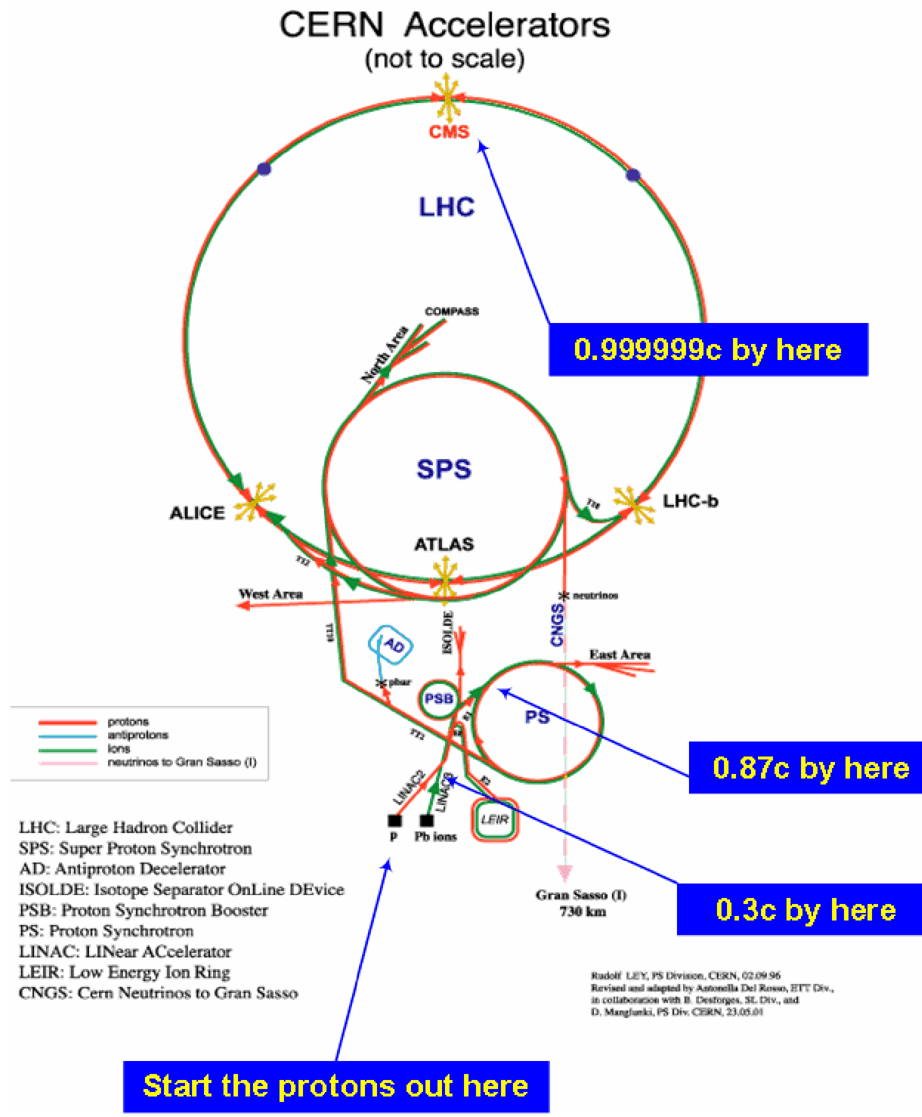




STAR
specialty: large acceptance measurement of hadrons

PHENIX
specialty: rare probes, leptons, and photons

The Large Hadron Collider



- 27 km circumference
- tunnel 3.8 m. in diameter, buried 50 to 175 m below ground straddles the French-Swiss border to the North-West of Geneva
- 1232 dipole magnets bend the beam
- Top energy: 14 TeV for pp, 5.5 TeV PbPb
- 4 detectors : ALICE, CMS, ATLAS, LHCb

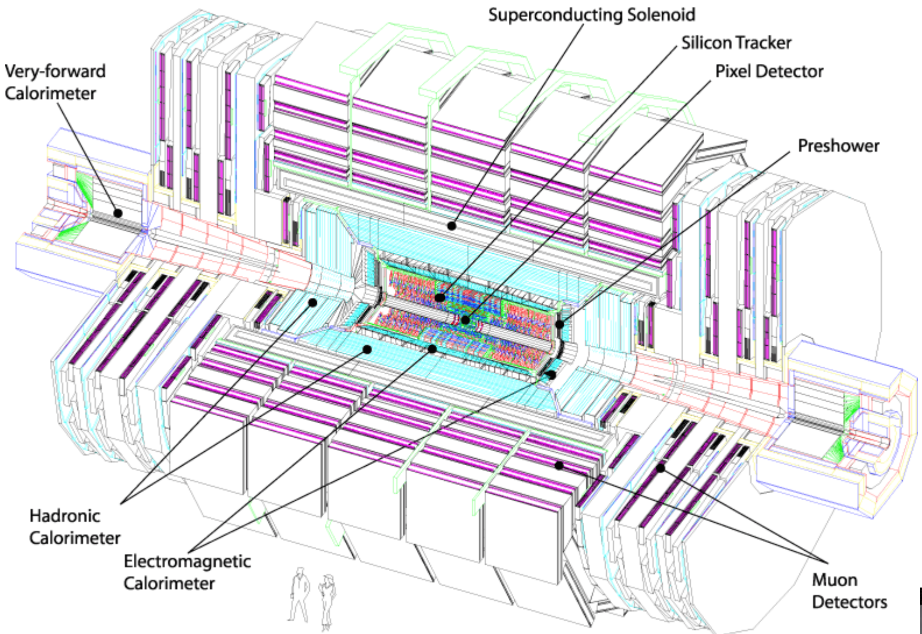
Some pictures from the LHC

A magnet going down the shaft.

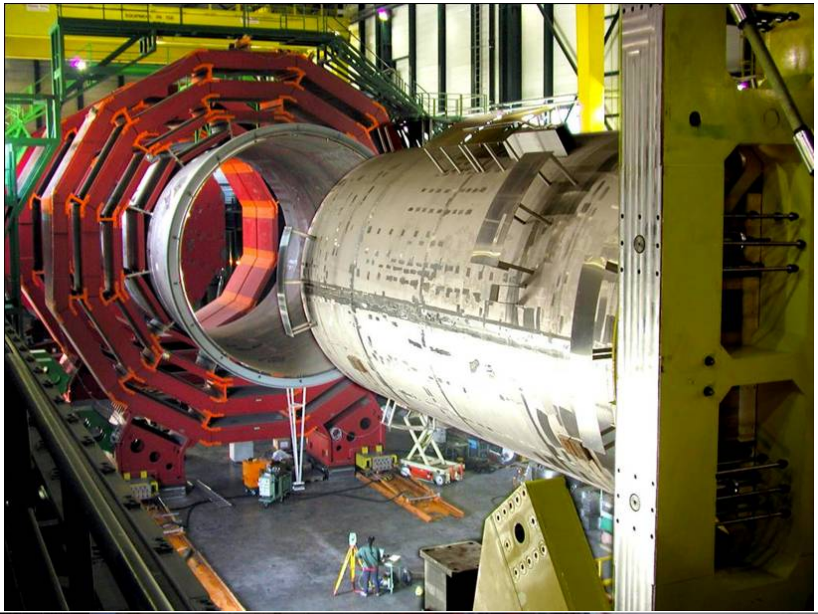
Inside the tunnel.

The (latest) start-up date is May 2008 !

The CMS detector



Compact Muon Solenoid



Natural units in high energy physics

$$\hbar = c = k_B = 1$$

- Energy, momentum, mass, temperature are all measured in energy units : MeV , GeV, TeV
- Hadron masses ~ 1 GeV
- Hadron sizes ~ 10^{-15} meters (aka 1 femtometer or 1 fermi = 1 fm)
- Nuclear sizes: R = 1.2 $A^{1/3}$ fm
  - For Au nucleus, A=197 , R ~ 7 fm
- Useful relation:

$$\hbar c = 197 MeV.fm \sim 200 MeV.fm$$

- 1 fm^{-1} ⇔ 200 MeV
- 200 MeV ~ characteristic scale associated with confinement
- distance and time are measured in inverse energy units

Four vectors for position and momentum

$$x_\mu = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} ct & x & y & z \end{pmatrix}$$

$$x_T \equiv x_\perp = \sqrt{x^2 + y^2}$$

$$x_\mu = \begin{pmatrix} t & x_T & z \end{pmatrix}$$

time, position

$$p_\mu = \begin{pmatrix} p_0 & p_1 & p_2 & p_3 \end{pmatrix} = \begin{pmatrix} \frac{E}{c} & p_x & p_y & p_z \end{pmatrix}$$

$$p_T \equiv p_\perp = \sqrt{p_x{}^2 + p_y{}^2}$$

$$p_\mu = \begin{pmatrix} E & p_T & p_z \end{pmatrix}$$

Energy, momentum

4-vector multiplication

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} .$$

$$x^{\mu} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$x_{\mu} = \begin{pmatrix} x_0 & x_1 & x_2 & x_3 \end{pmatrix} = g_{\mu\nu} x^{\nu} = \begin{pmatrix} x^0 & -x^1 & -x^2 & -x^3 \end{pmatrix}$$

$$a.b = a_{\mu} b^{\mu} = g_{\mu\nu} a^{\nu} b^{\mu} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - \vec{a}.\vec{b}$$

We will often use:

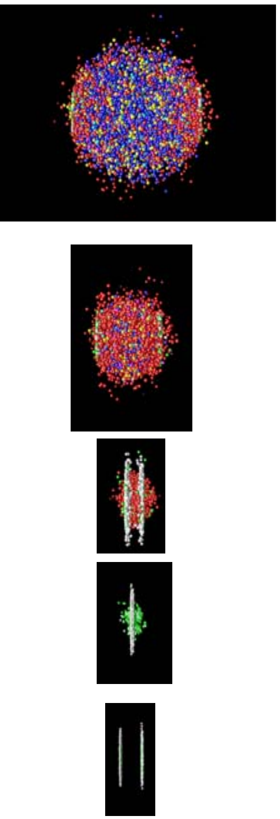
$$\mathbf{p} \cdot \mathbf{x} = Et - \vec{p} \cdot \vec{x}$$

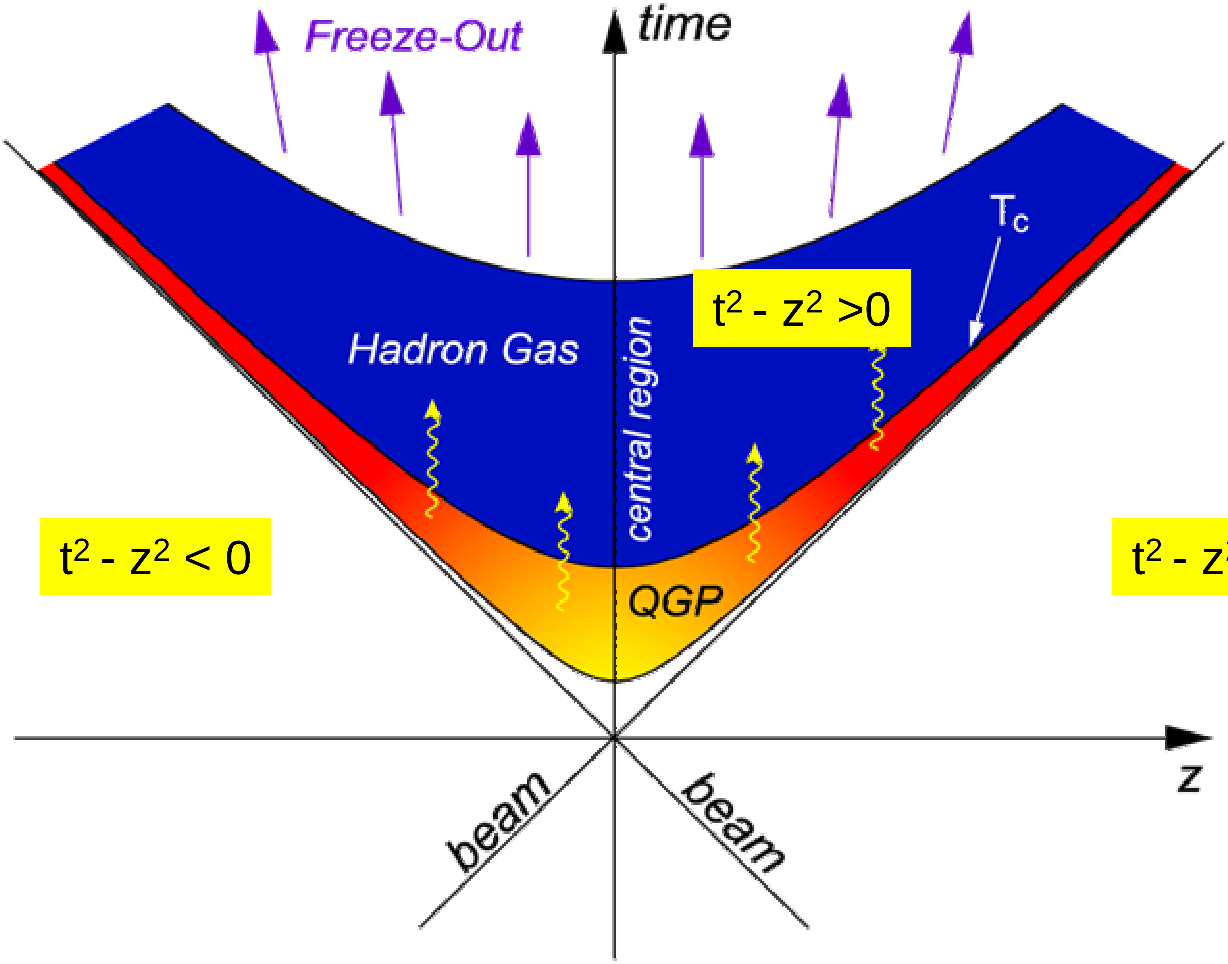
$$\mathbf{p}_1 \cdot \mathbf{p}_2 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$$

When 1 = 2 ,

$$\mathbf{p} \cdot \mathbf{p} = \mathbf{p}^2 = E^2 - |\vec{p}|^2 = m^2$$

Space-time picture of a nuclear collision





Fixed target and collider experiments

- Accelerate one beam and aim it at a stationary target: “fixed target”. We refer to the coordinate system in which one nucleus is moving and the other is stationary as the “laboratory system” or “laboratory frame”. In this case the c.m. system is moving with respect to the lab system
- Collider: 2 beams (could be the same species AA, pp, or different species pA, dA, AB)
- When AA and both beams have the same energy in a collider – the center of mass is stationary (lab=c.m.) Not so for asymmetric species.
- Need to transform between different coordinate systems
- Some conventions and notation:
  - Beam is along z axis
  - Machine parameters usually give energy-per-nucleon. This is kinetic energy. The total energy is $E = E_{kin} + m$
  - Note: Make sure that you distinguish between the velocity of a particle and the velocity of the reference frame !
  - Velocity of a particle moving along the z axis: $\beta = p_z/E$
  - The Lorentz factor for a particle moving along the z axis with velocity β:

$$\gamma = (1-\beta^2)^{-1/2} = E/\sqrt{E^2 - p_z^2} = E/m$$

Lorentz transformations

$$\begin{pmatrix} E^* \\ p_z^* \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}$$

Moving frame

lab

- Frame F^* with respect to frame F with velocity β along the z –axis
- The energy and momentum of a particle transform as shown above.
- What happens to p_T ?
- Define a new variable (we'll see later why it is convenient)

$$m_T^2 = E^2 - p_z^2 = p_T^2 + m^2$$

- Prove that m_T is Lorentz invariant i.e. – it does not change under Lorentz transformation)

Transverse mass and mass

$$\begin{pmatrix} E^* \\ p_z^* \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}$$

$$\begin{aligned} E^* &= \gamma(E - \beta p_z) \\ p_z^* &= \gamma(p_z - \beta E) \end{aligned}$$

$$m_T^* = E^{*2} - p_z^{*2} = \gamma^2(E^2 + \beta^2 p_z^2 - p_z^2 - \beta^2 E^2) =$$

$$= \gamma^2(1 - \beta^2)(E^2 - p_z^2) = m_T$$

- How does mass transform ?

Rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

$$y = \frac{1}{2} \ln \frac{1 + \beta}{1 - \beta}$$

Velocity of particle

- Why is it useful ?
- Velocities transformations are not linear
- Rapidity is additive: We go from a frame *F* in which the rapidity of the particle is *y* to a frame F^* moving with respect to *F* with velocity β_f

$$y^* = \frac{1}{2} \ln\left(\frac{E^* + p^*_z}{E^* - p^*_z}\right) = y - \frac{1}{2} \ln\left(\frac{1+\beta_f}{1-\beta_f}\right) = y - y_f$$

Velocity of frame

Rapidity transformations: proof

$$\begin{aligned} y^* &= \frac{1}{2}\ln\left(\frac{E^* + p_z^*}{E^* - p_z^*}\right) = \frac{1}{2}\ln\left(\frac{\gamma(E - \beta p_z) + \gamma(p_z - \beta E)}{\gamma(E - \beta p_z) - \gamma(p_z - \beta E)}\right) \\ &= \frac{1}{2}\ln\left(\left(\frac{1-\beta}{1+\beta}\right)\left(\frac{E+p_z}{E-p_z}\right)\right) = y - \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right). \end{aligned}$$

- Note: beta and gamma are the velocity and the gamma factor of the moving frame. I skipped the indices.

Another way to look at rapidity

Start from definition:

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \ .$$

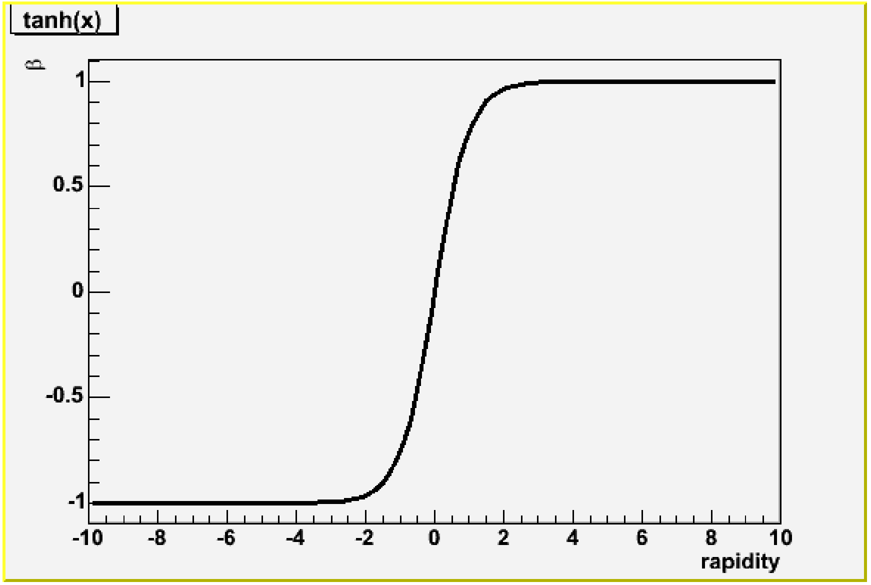
$$\sinh y = [\exp(y) - \exp(-y)]/2$$
$$\cosh y = [\exp(y) + \exp(-y)]/2.$$

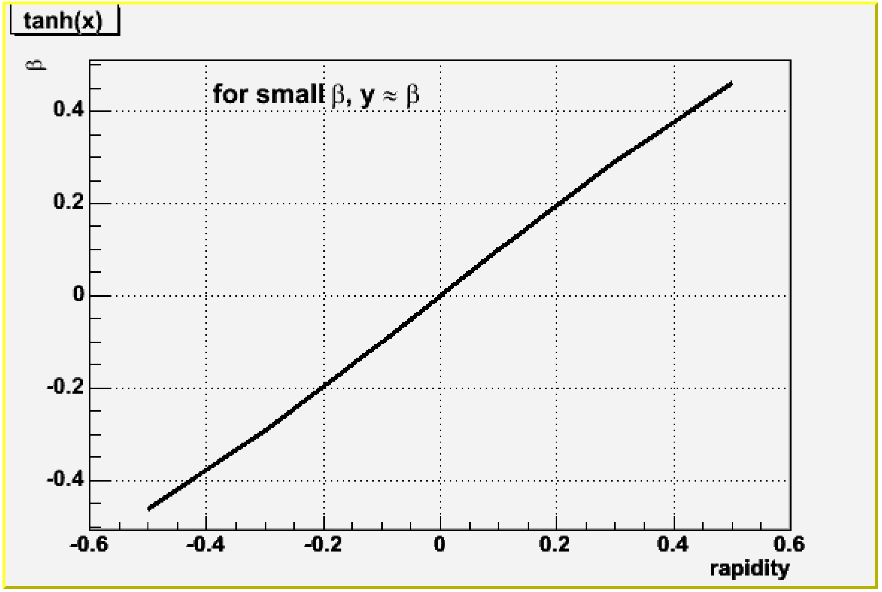
$$\begin{aligned} E &= m_T \cosh y \\ p_z &= m_T \sinh y \end{aligned}$$

With $\beta = p_z/E$ →

$$\beta = \tanh y$$

Beta and rapidity ; pseudo-rapidity

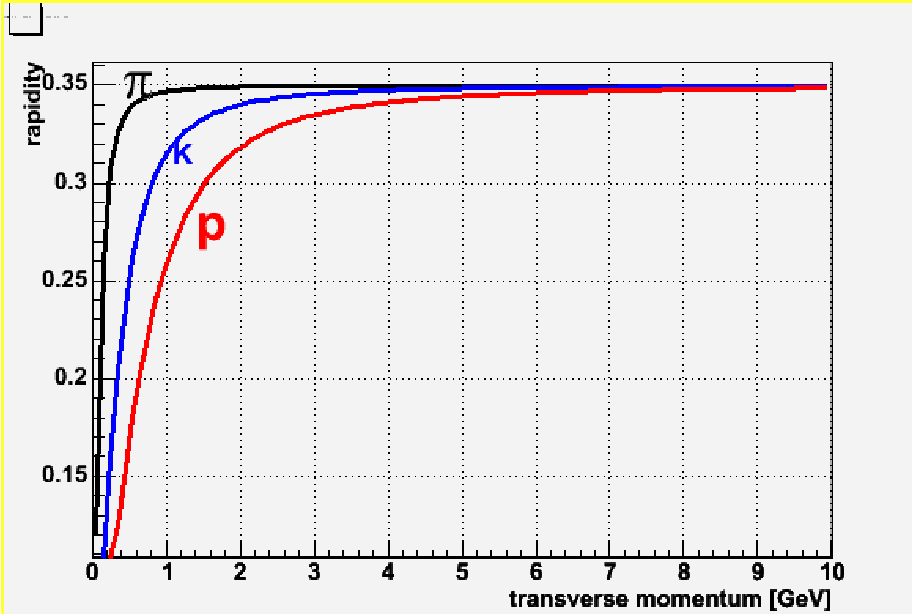




- For small β , y~β
- For p>>m, y~ η – pseudo-rapidity

$$y \approx -\ln[\tan(\theta/2)] \equiv \eta$$

Pseudo-rapidity, rapidity and acceptance



$$\eta = \frac{1}{2}\ln\left(\frac{p+p_z}{p-p_z}\right)$$

- Example: for PHENIX experiment – not full acceptance . $110^o > \theta > 70^o \Rightarrow \eta = +/-0.35$
- But rapidity depends on mass at small momentum:
  - Different acceptance in rapidity for different particle species

- Next time: continue with Lorentz invariants and cross-section
- Homework will be assigned after next lecture.

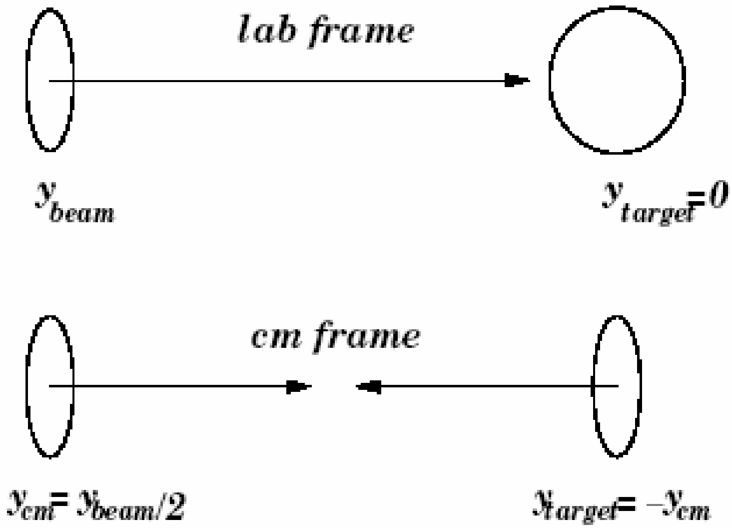


Figure 1.5: The rapidities are shown for the laboratory (top) and center-of-mass (bottom) frames. In the laboratory frame, the projectile is shown as Lorentz contracted while the target is not. In the center-of-mass frame, both the projectile and target are boosted.