Lecture 2: Invariants, cross-section, Feynman diagrams

- Last lecture introduced:
  - 4-vectors
  - 4-vector multiplication
  - Lorentz transformations
  - Introduced : transverse mass, transverse momentum , rapidity, pseudo-rapidity
  - Rapidity transformation with Lorentz boost
  - Showed that m_T, m , p_T are Lorentz invariant

Today's lecture

- Now that we know the appropriate language to describe the kinematics, we want to move to a more exciting things like collisions (i.e. interactions) between particles
- We start with AA collisions in the initial state and detect particles:π,K,p, γ,e^+,e^-,μ^+,μ^- in the final state
- We can reconstruct particles that have decayed: Λ,J/ψ,ϕ,Ω...
- We want to know if AA is just a superposition of NN or if we have created a QGP medium that affects the particle production
- => we need to understand NN collisions first

Two main processes to consider

- Scattering: 2->2 and 2-> many
- Decay: 1->2 ; 1-> several
- We need to describe the production rate of particles in the final state in a frame-independent way
- Cross-section : has dimension of area . Measured in barns = 10^{-24} cm^2

Cross-section

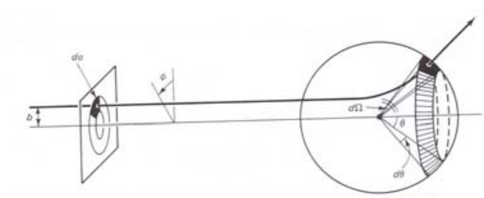


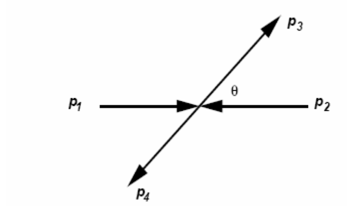
Figure 6.3 Particle incident in area $d\sigma$ scatters into solid angle $d\Omega$.

$$L = \frac{fnN_1N_2}{A}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{L}\frac{d^2N}{d\Omega dt}$$

- Cross-section: particles coming through area $d\sigma$ scatter in $d\Omega$
- Luminosity:
  - *f* is the revolution frequency
  - *n* is the number of bunches in one beam in the storage ring.
  - N_i is the number of particles in each bunch
  - *A* is the cross section of the beam.
  - RHIC design luminosity: 2×10^{26} cm^{-2} s^{-1}
  - Currently: $\sim 5\times 10^{26}$ cm^{-2} s^{-1}

2-> 2 scattering and invariants



- Particle (1) and (2) in the initial state
- Particle (3) and (4) in the final state
- p_1, p_2, p_3, p_4 – 4-momenta

Mandelstam invariants:

$$
\begin{aligned}
S &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
T &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\
U &= (p_1 - p_4)^2 = (p_2 - p_3)^2
\end{aligned}
$$

Do this on the board or for homework

- Show that S is Lorentz invariant
- Show that sqrt(S) = E_{cm}
- Find the relationship between the beam momentum p_1 in a fixed target experiment and the center of mass momentum
- Show that :

$S+T+U = \Sigma_i m_i^2$

- Show that d^3p/E is invariant

The Golden rule

- Cross section and decay rates are expressed in terms of:
  - The amplitude for the process: *M* (or matrix element between the initial and final state).
    - Contains all dynamical information
    - Calculated by evaluating appropriate Feynman diagrams using the Feynman rules for the appropriate interaction in question
  - The available phase space (or density of final states) – contains the kinematical information
  - Example: decay of a heavy particle into many light secondaries – large phase space – many different ways to apportion the available energy. Neutron decay – small phase space, because m_p is close to m_n

$$transition \quad rate = \frac{2\pi}{\hbar}\left|M\right|^2 \times phase \quad space$$

Golden rule (II)

- 1->2+3+….+n

$$dLips = (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - \ldots - p_n)\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\ldots\frac{d^3p_n}{(2\pi)^3 2E_n}$$

$$d\Gamma = \left|M\right|^2 \frac{dLips}{2m_1}$$

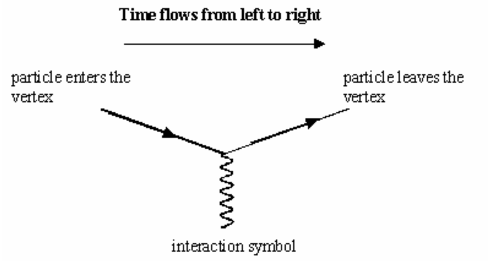
- 1+2->3+4+..n

$$dLips = (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4 \ldots - p_n)\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\ldots\frac{d^3p_n}{(2\pi)^3 2E_n}$$

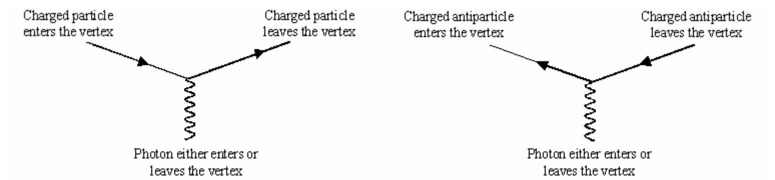
$$d\sigma = \left|M\right|^2 \frac{dLips}{initial \quad flux}$$

What goes into M ?

- Here we need to consider the interaction
  - Note: a vertex is simply a symbol, it does not represent tracks of particles in space and it is not a space-time diagram
  - At each vertex, energy and momentum is conserved
  - At each vertex, we put the coupling constant , i.e. -> the strength of the interaction

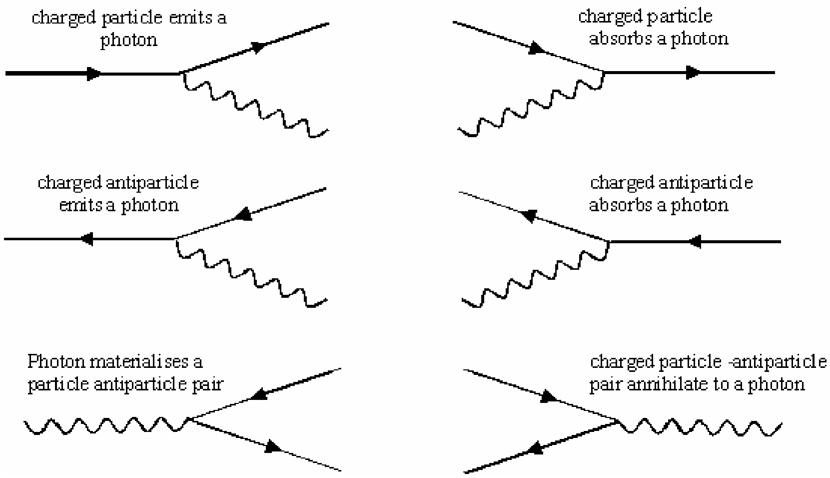


Electromagnetic interaction



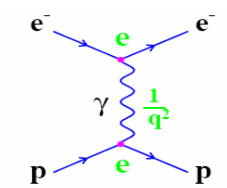
- The coupling is sqrt(α) , $\alpha = e^2/4\pi \sim 1/137$

QED processes: rotate the legs around

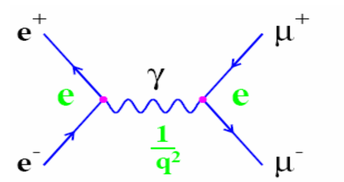


QED - continued

electron-proton scattering

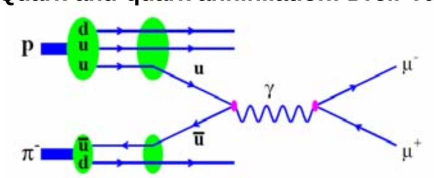


electron-positron annihilation



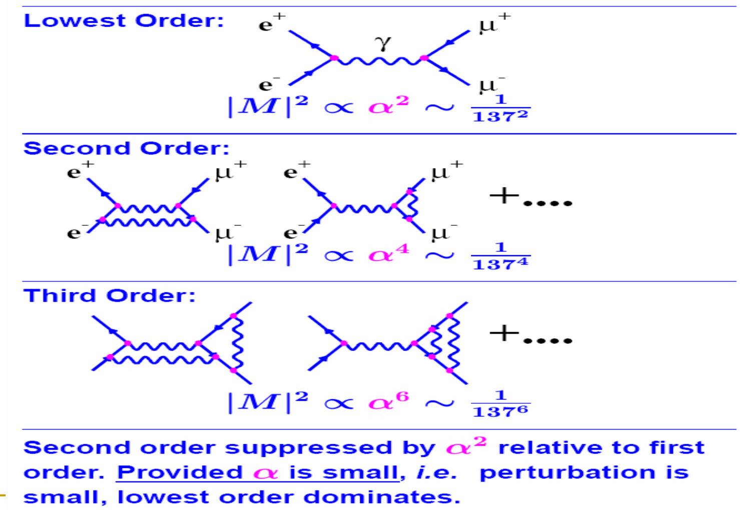
$$\pi^- p \rightarrow \mu^+ \mu^- + hadrons$$

Quark-anti-quark annihilation: Drell-Yan

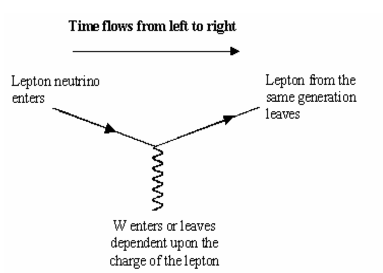


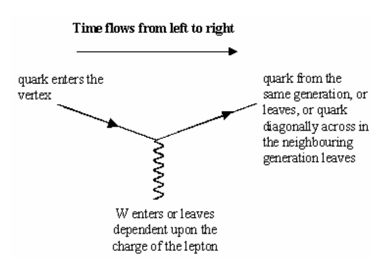
M ~> is expressed I the same way ~ e^2/q^2 However, the four-momentum transfer is very different

Perturbative expansion



Weak vertices

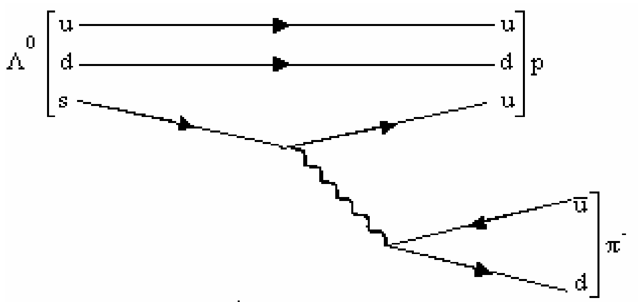


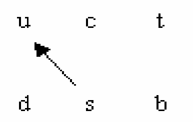


- We could also exchange a Z boson: any quark or lepton doesn't change identity
- Additional vertices: W couples to Z, W and to photon

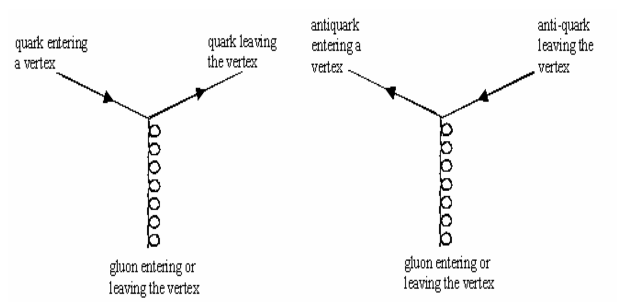
•NOTE: the mass of the gauge boson goes in the denominator of the propagator. That's what makes the interaction "weak". The coupling constant is actually larger Than the EM coupling constant.

Weak decay example: quarks change generation





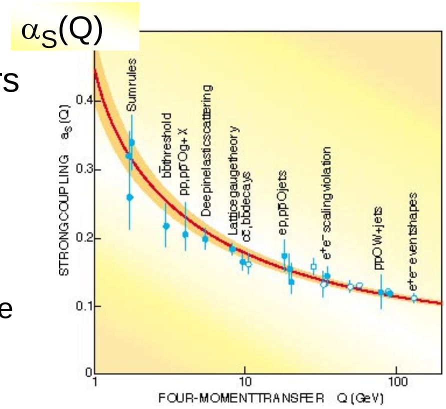
Strong interaction



Gluons carry color+anti-color. They couple to themselves and to quarks.
The interaction is strong due to the coupling constant being ~ 1.
Higher order diagrams contribute more ☹ . Can't do a perturbative expansion!
We get one lucky break. The coupling constant is NOT a constant !

QCD's Key Feature

- Hadron masses ~ 1 GeV
- Hadron sizes ~ 10^{-15} meters (aka 1 femtometer or 1 fermi = 1 fm)
- Planck's constant
  $\hbar c$ = 0.2 GeV-fm
  - ➔ 1 fm^{-1} ⇔ 200 MeV
  - ➔ 200 MeV ~ characteristic scale associated with confinement
- These are reflected in the "running coupling constant" of QCD
- Can do pQCD when Q^2 is large "hard scattering"



0.2 fm 0.02 fm 0.002 fm

Cross-sections in pp collisions

- Total = elastic + inelastic
- Elastic ~ 10 mb
- Inelastic ~ 40 mb at RHIC energies
- Energy dependence : parameterized from data

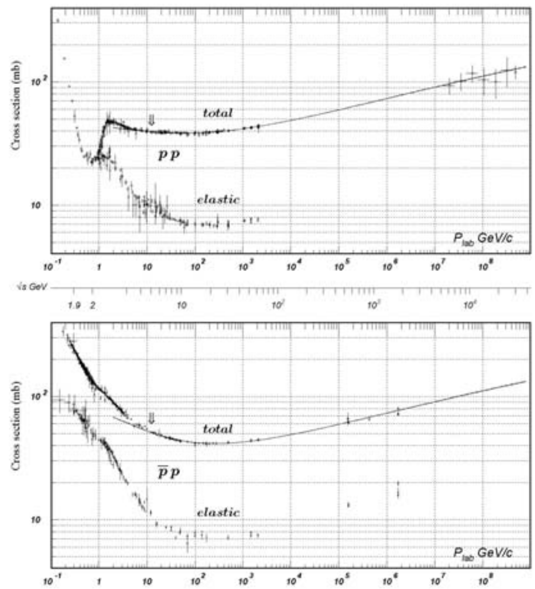


Figure 40.11: Total and elastic cross sections for pp and $\overline{p}p$ collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/xsect/contents.html` (Courtesy of the COMPAS group, IHEP, Protvino, August 2005)