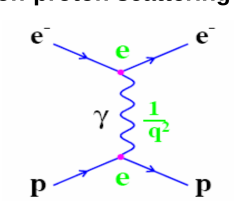
Lecture 3:

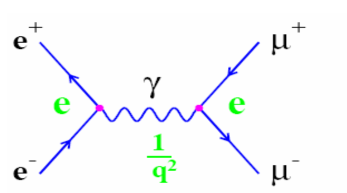
- Last time:
  - Invariants
  - Cross-section: Fermi's golden rule
  - Feynman diagrams (rules) for QED, weak, QCD
- Today: electron-proton scattering and the evidence for nucleon structure

2 QED processes to consider

electron-proton scattering

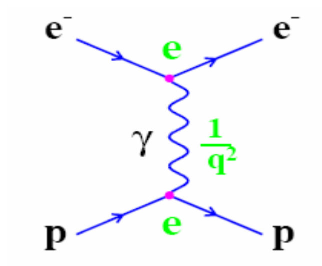


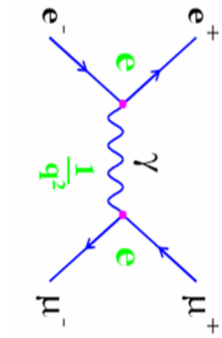
electron-positron annihilation



***M* ~> is expressed I the same way ~ e^2/q^2 However, the four-momentum transfer is very different.**

What if I take the second diagram and rotate it . Will these be the same now ?





No, not really. The proton is not a point charge.

Mini-history of scattering experiments

- Rutherford, Geiger and Marsden direct alpha particles into a gold foil, and detect the number which pass through to the other side.
- "Plum-pudding" model predicts only very small angle scattering; nothing at large angles
- Large angle scattering observed!
- Consistent with scattering from a point charge much smaller than size of atom: the nucleus!
  - *Scattering by electromagnetic force which is well understood theoretically (reliable prediction).*
  - *The angular distribution of events from a point charge is known as the Mott cross section (relativistic) or Rutherford (non-relativistic).*

Electron-Nucleus Scattering

Hofstadter *et al.* use an electron beam at Stanford to study the atomic nucleus (and nucleon) further.

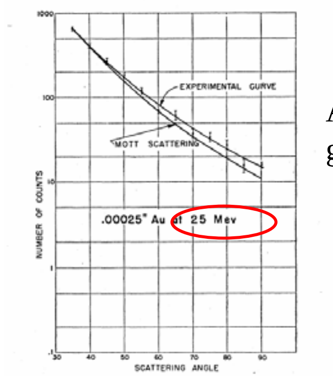


FIG. 8. The angular distribution of scattered electrons from a gold foil, ¼-mil thick, at 25 Mev. The foil plane was at 45° with respect to the beam. The Mott curve for a point charge is shown. Arbitrary normalization is made at 35°.

At low energy (large wavelength), gold nucleus looks like a point charge

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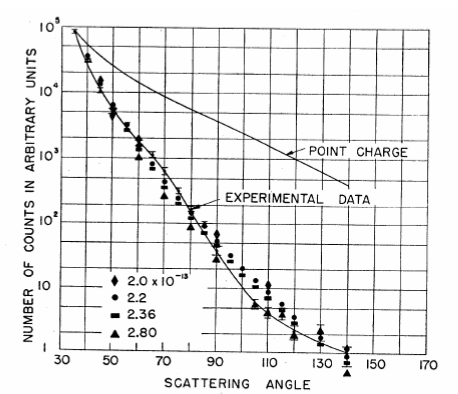


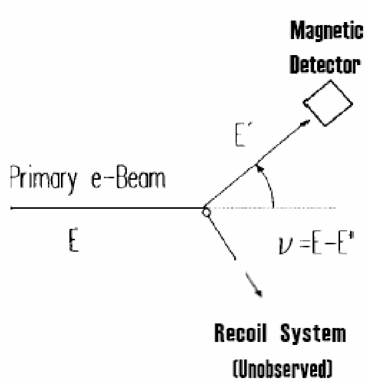
FIG. 11. The angular distribution of electrons scattered from a 2-mil gold foil at 125 Mev. The point charge calculation of Feshbach is indicated. Theoretical points based on the first Born approximation for exponential charge distributions are shown. Values of $\alpha = 2.0, 2.2, 2.36, 2.8 \times 10^{-13}$ cm are chosen to demonstrate the sensitivity of the angular distribution to change of radius. All curves are normalized arbitrarily at 35°.

At "high" energy, there is deviation from the point charge prediction!

The nucleus is not a point charge if we look with enough resolution!

Now go to SLAC and find out what happens in e-p scattering

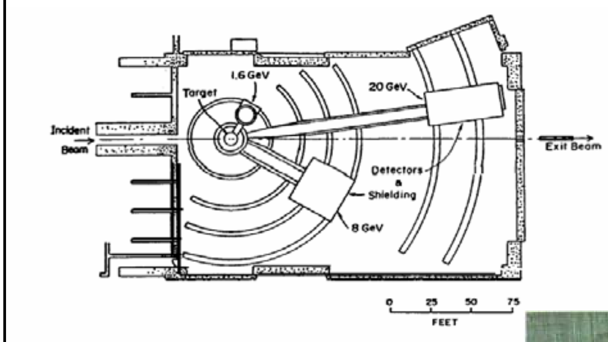
Kinematics of e-p scattering



$$\nu = E\text{-}E' = q^2/(2M) \qquad q^2 = 2EE'\ (1\text{-}\cos\theta)$$

Energy loss — Momentum transfer (neglect m_e)

The MIT-SLAC experiment



Elastic scattering: ee and ep

For e-e or e-μ: the Mott cross section can be calculated using Feynman calculus for QED. Scattering of point objects. In the non-relativistic in the limit $p^2 << m_e^2$, Mott reduces to Rutherford .

$$\sigma_M = \frac{4\alpha^2 E'^2}{q^4} \cos^2\left(\frac{\theta}{2}\right)$$

For ep : the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega}(E) = \sigma_M(E) \left\{\frac{E'}{E}\right\} \left\{ \frac{G_{Ep}^2(q^2) + \tau G_{Mp}^2(q^2)}{1+\tau} + 2\,\tau\, G_{Mp}^2 \tan^2(\theta/2) \right\}$$

$$\tau = q^2/(4M^2) \qquad G_{Ep}(q^2) \text{ and } G_{Mp}(q^2),$$

The electric and magnetic form factors describe the time-averaged structure of the proton. In the non-relativistic limit the squares of these functions are the Fourier transforms of the spatial distributions of charge and magnetic moment, respectively. They depend only on 1 parameter (q^2 for theorists and θ, for experimentalists.)

Elastic scattering results

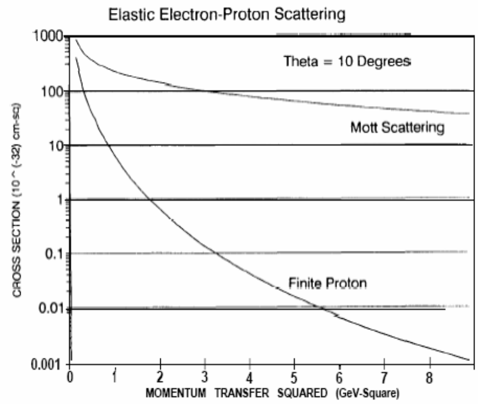


Fig. 4. Elastic scattering cross sections for electrons from a "point" proton and for the actual proton. The differences are attributable to the finite sire of the proton.

$$G_{Mp}/\mu = 1/(1 + q^2/0.71)^2$$

Here μ=2.79 is the proton's magnetic moment. Measurements for the form factor agree with a phenomenological dipole function up to ~ q^2~ 10 GeV^2

Inelastic e-p scattering. The proton breaks-up into many particles.

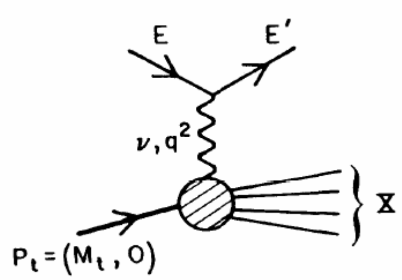


Fig. 5. Feynman diagram for inelastic electron scattering.

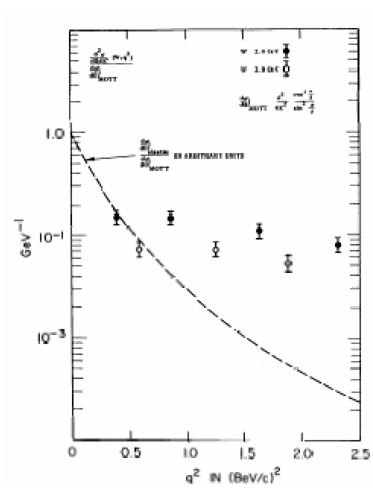
Note: in e-p elastic, E' is kinematically determined by E and θ. However, here the total hadronic momentum squared does not need to equal M^2, as it would be for a single proton.

Inelastic x-section and structure functions

$$\frac{d^2\sigma}{d\Omega\, dE'}(E,E',\theta) = \sigma_M \left[W_2(\nu,q^2) + 2W_1(\nu,q^2)\tan^2(\theta/2)\right]$$

•This expression is the analog of the Rosenbluth cross section
•The inelastic structure functions depend on 2 variables (ν,q^2)
•The structure functions W_1 and W_2 are similarly defined for the proton, deuteron, or neutron; they summarize all the information about the structure of the target particles obtainable by scattering unpolarized electrons from an unpolarized target. (i.e. the spin is not defined).

The data:



- Large inelastic x-sect and flat with q^2

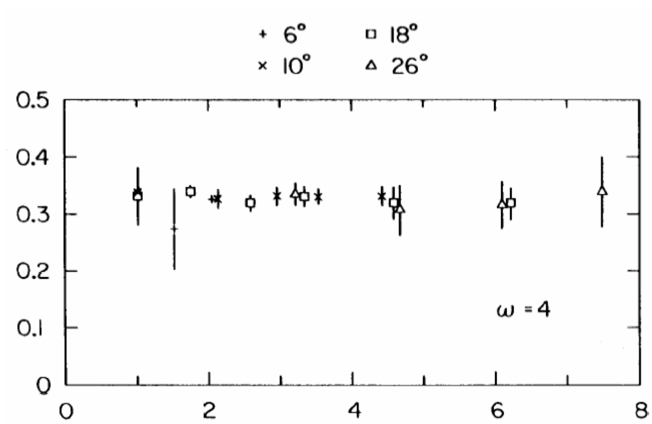
Scaling and scaling variables

- Bjorken conjectured that in the limit of q^2 and ν approaching infinity, with the ratio $\omega = 2M\nu/q^2$ held fixed, the two quantities νW_2 and W_1 become functions of ω only.

$$2MW_1(\nu, q^2) = F_1(\omega)$$
$$\nu W_2(\nu, q^2) = F_2(\omega)$$

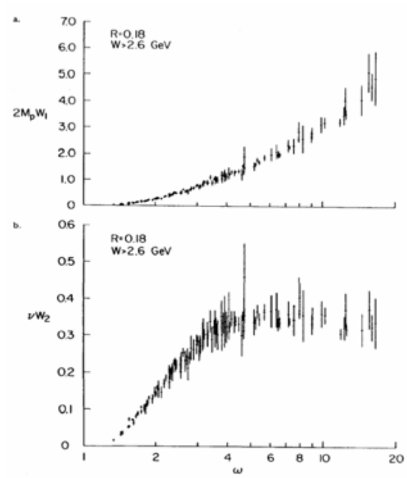
- Later, the quantity $1/\omega = x$ was used and identified as the fraction of the proton momentum (mass) carried by the struck quark
- Scaling has been shown to be a consequence of the fact that the proton consists of point-like constituents
- Spin ½ of the constituents dictates that $2xF_1(x) = F_2(x)$

Scaling of structure functions. The figure shows νW_2.



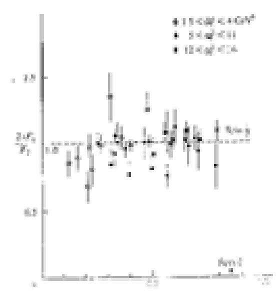
ı νW_2, for ω = 4, as a function of q^2

More scaling data



The spin of the constituents: Callan-Gross relation

$2xF_1 / F_2$



x

Where are the constituents?

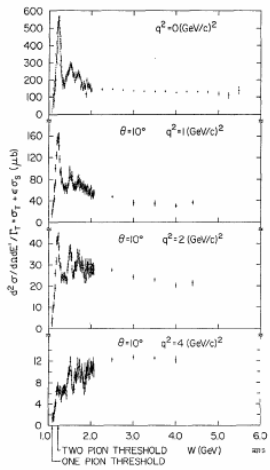


FIGURE 9. Spectra of electrons scattered from hydrogen at momentum transfers squared up to 4 (GeV/c)². The curve for $q^2=0$ represents an extrapolation to $q^2=0$ of electron scattering data acquired at $\theta=1.5°$ (18).

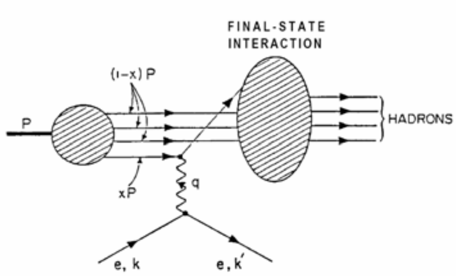
At high energy loss and small wavelength, we so only a flat featureless spectrum. This is the signature of scattering from point-like objects!

Initially these point-like objects are called partons

Sum rules and the constituents

- If the proton with momentum p is made up of partons each carrying some fraction x_i of the proton momentum and charge Q_i:

$$\nu W_2(\nu, q^2) = \sum_N P(N) \left(\sum_{i=1}^{N} Q_i^2 \right) x f_N(x) = F_2(x)$$



P(N) is the probability of N partons occurring.

f N(x) is the distribution of the longitudinal momenta of the charged partons

The data showed that the protons are not just u and d quarks

– **About half of the nucleon momentum is carried by gluons**

Parton distribution functions