Lecture 10 : Statistical thermal model

- Hadron multiplicities and their correlations and fluctuations (event-by-event) are observables which can provide information on the nature, composition, and size of the medium from which they are originating.
- Of particular interest is the extent to which the measured particle yields are showing thermal equilibration. Why ?
- We will study:
  - particle abundances: chemical composition
  - Particle momentum spectra: dynamical evolution and collective flow
- Statistical mechanics predicts thermodynamical quantities based on average over stat ensemble and observing conservation laws.

Statistical approach

- The basic quantity required to compute the thermal composition of particle yields measured in heavy ion collisions is the partition function Z(T, V). In the Grand Canonical (GC) ensemble,

$$Z^{GC}(T, V, \mu_Q) = \mathrm{Tr}[e^{-\beta(H - \sum_i \mu_{Q_i} Q_i)}]$$

where H is the Hamiltonian of the system, Q_i are the conserved charges and μ_{Qi} are the chemical potentials that guarantee that the charges Q_i are conserved on the average in the whole system. β is the inverse temperature.

- The GC partition function of a hadron resonance gas can then be written as a sum of partition functions $\ln Z_i$ of all hadrons and resonances

$$\ln Z(T, V, \vec{\mu}) = \sum_i \ln Z_i(T, V, \vec{\mu})$$

$$\ln Z(T,V,\vec{\mu}) = \sum_i \ln Z_i(T,V,\vec{\mu}), \tag{4}$$

where $\epsilon_i = \sqrt{p^2 + m_i^2}$ and $\vec{\mu} = (\mu_B, \mu_S, \mu_Q)$ with the chemical potentials μ_i related to baryon number, strangeness and electric charge, respectively. For particle i of strangeness S_i, baryon number B_i, electric charge Q_i and spin–isospin degeneracy factor g_i,[2]

$$\ln Z_i(T,V,\vec{\mu}) = \frac{Vg_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\beta\epsilon_i)], \tag{5}$$

with (+) for fermions, (-) for bosons and fugacity

$$\lambda_i(T,\vec{\mu}) = \exp(\frac{B_i\mu_B + S_i\mu_S + Q_i\mu_Q}{T}) \tag{6}$$

Expanding the logarithm and performing the momentum integration in Eq. (5) we obtain

$$\ln Z_i(T,V,\vec{\mu}) = \frac{VTg_i}{2\pi^2} \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} \lambda_i^k m_i^2 K_2(\frac{km_i}{T}), \tag{7}$$

where K_2 is the modified Bessel function and the upper sign is for bosons and lower for fermions. The first term in Eq. (7) corresponds to the Boltzmann approximation. The density of particle i is obtained from Eq. (7) as

$$n_i(T,\vec{\mu}) = \frac{\langle N_i \rangle}{V} = \frac{Tg_i}{2\pi^2} \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \lambda_i^k m_i^2 K_2(\frac{km_i}{T}), \tag{8}$$

The statistical model parameters

- The partition function (and its derivatives) depends in general on five parameters. However, only three are independent, since the isospin asymmetry in the initial state fixes the charge chemical potential and the strangeness neutrality condition eliminates the strange chemical potential.
- Thus, on the level of particle multiplicity ratios we are only left with temperature T and baryon chemical potential μ_B as independent parameters.
- If we find agreement between the statistical model prediction and data: the interpretation is that this implies statistical equilibrium at temperature T and chemical potential μ_B. Statistical equilibrium is a necessary (but not sufficient) condition for QGP formation.

Statistical "penalty" factors and associated production

$$R_p \approx \exp\frac{m \pm \mu_b}{T}$$

$$\langle N_{K^+}\rangle_{eq} = \langle N_{K^-}\rangle_{eq} + \langle N_\Lambda\rangle_{eq}$$

- Sign in μ_b
- - for matter
- + for anti-matter
- μ_b ~ 450 MeV at AGS
- μ_b ~ 30 MeV at RHIC in central rapidity
- Associated production:
  - NN-> $N\Lambda K^+$
    - $Q = m_\Lambda + m_K - m_N$ = 672 MeV
  - NN->NNK^+K^-
    - $Q=2\ m_K$ = 988 MeV

What else appears in models: strangeness is special !

- Sometimes an additional factor γ_s (<=1) is needed to describe the data involving strange particles (we'll have a separate lecture on strangeness production)
- this implies a state in which strangeness is suppressed compared to the equilibrium value => additional dynamics present in the data which is not contained in the statistical operator and not consistent with uniform phase space density.
- Reminder: in small and cold systems strangeness is not copiously produced, thus we need to take care that it is absolutely conserved (not just on the average) and use a canonical partition function. If, in this regime, canonically calculated particle ratios agree with those measured, this implies equilibrium at temperature T and over the canonical volume V.
    - How do we know the volume ??

Comparison to model

$$\chi^2 = \sum_i \frac{(\mathcal{R}_i^{\mathrm{exp.}} - \mathcal{R}_i^{\mathrm{model}})^2}{\sigma_i^2}.$$

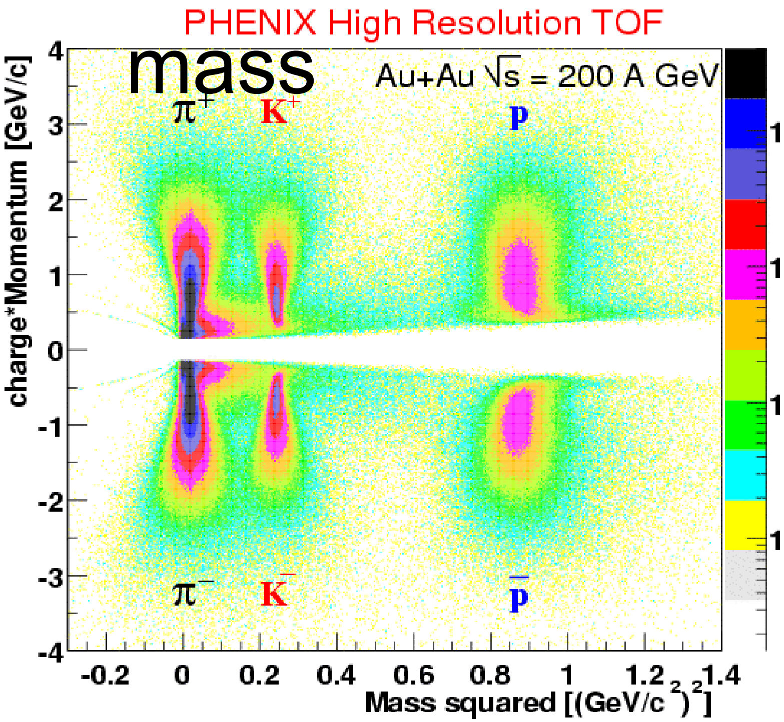
- The criterion for the best fit of the model to data is a minimum in χ^2
- Here: R_{model} and R_{exp} are the i^{th} particle ratio as calculated from the model or measured in the experiment
- σ_i represent the errors (including systematic errors where available) in the experimental data points as quoted in the experimental publications.

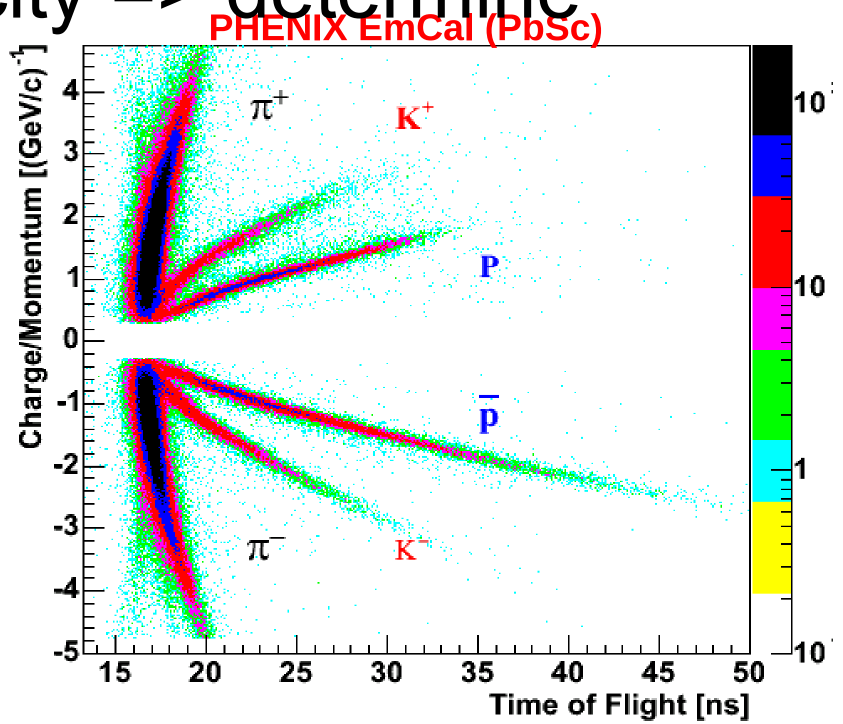
How do we measure the particle yields ?

- Identify the particle (by its mass and charge)
- Measure the transverse momentum spectrum
- Integrate it to get the total number of particles
- In fixed target experiment – everything goes forward (due to cm motion) – easy to measure total (4π) yield
- In collider experiment: measure the yield in a slice of rapidity : dN/dy
- Apply corrections for acceptance and decays

Methods for PID: TOF

- Time of flight measurement: measure momentum and velocity => determine mass





Time of Flight

- π/K separation ~ 3 GeV/c
- K/p separation ~ 5 GeV/c

σ_t ~115 ps

Electromagnetic Calorimeter

- π/K separation ~ 1 GeV/c
- K/p separation ~ 2 GeV/c

σ_t ~400 ps

PHENIX high-p_T detector

- Combine multiple detectors to get track-by-track PID to p_T ~ 9 GeV/c
-

		Pion-Kaon separation	Kaon-Proton separation
TOF	σ~100 ps	0 - 2.5	- 5
RICH	n=1.00044 γth~34	5 - 17	17 -
Aerogel	n=1.01 γth~8.5	1 - 5	5 - 9

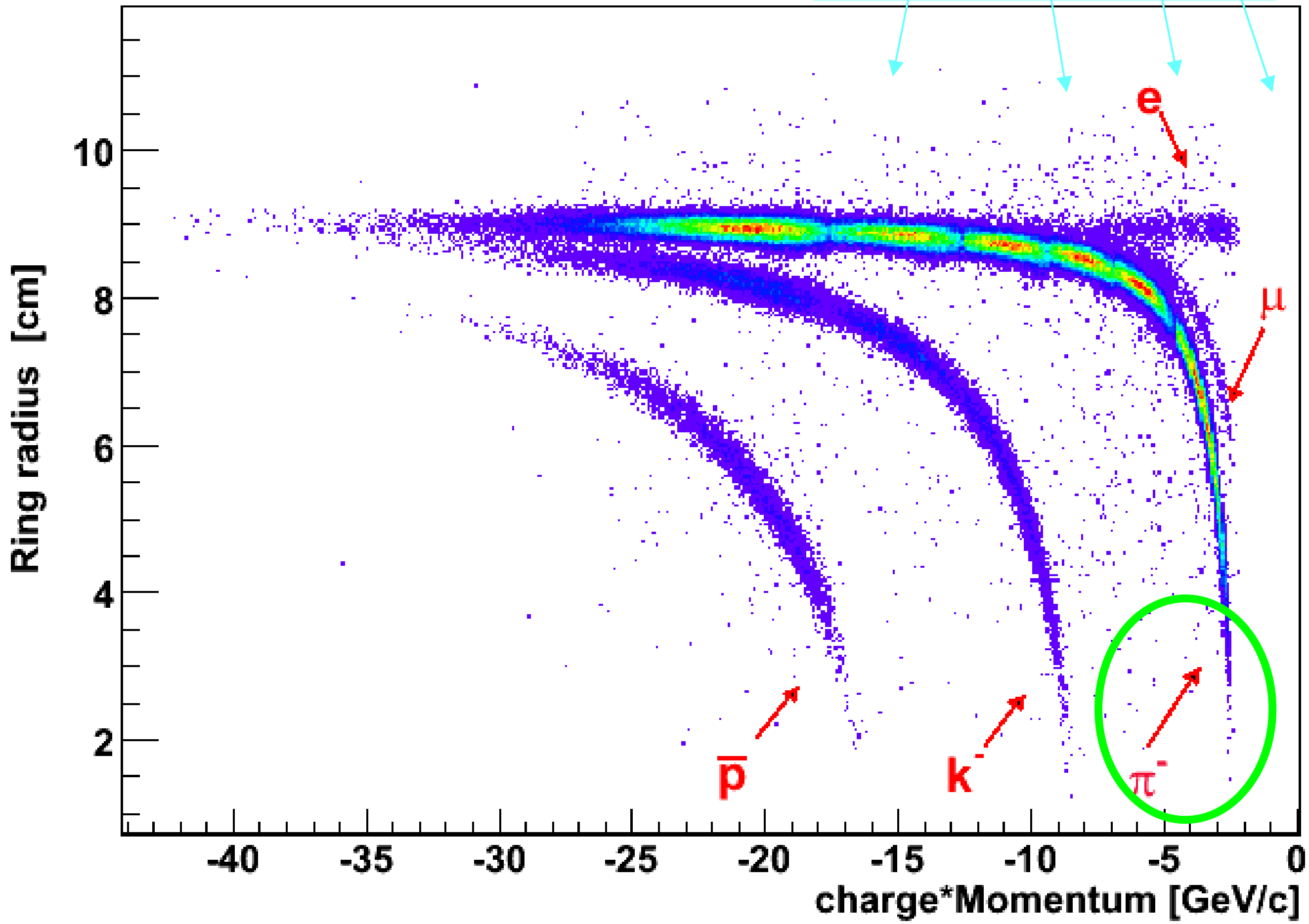
PID using Cere ectors

FS PID using RICH

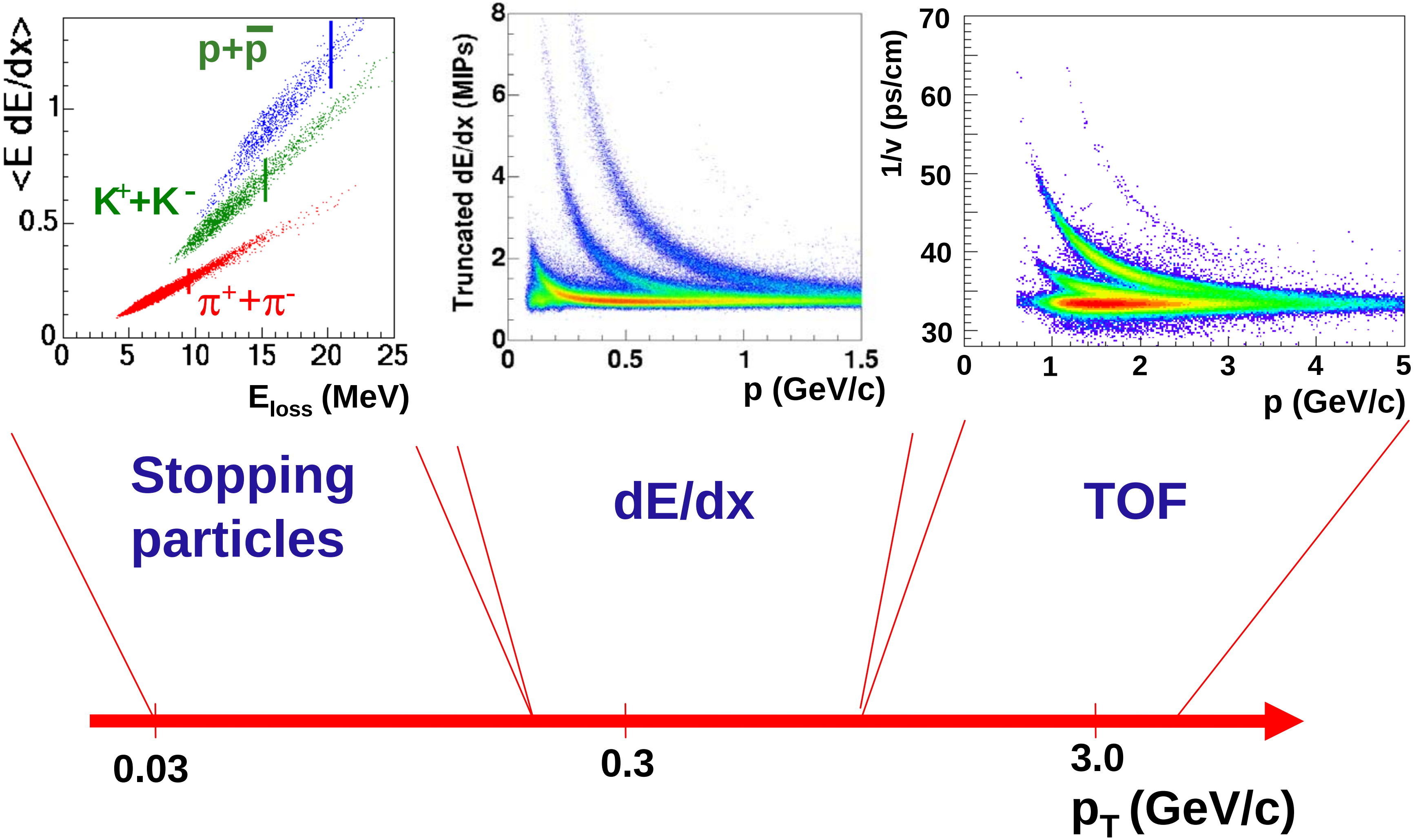
BRAHMS

Ring radius vs p

Multiple settings

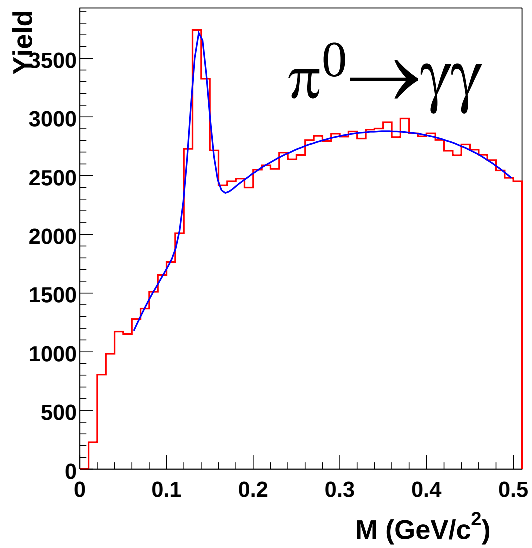


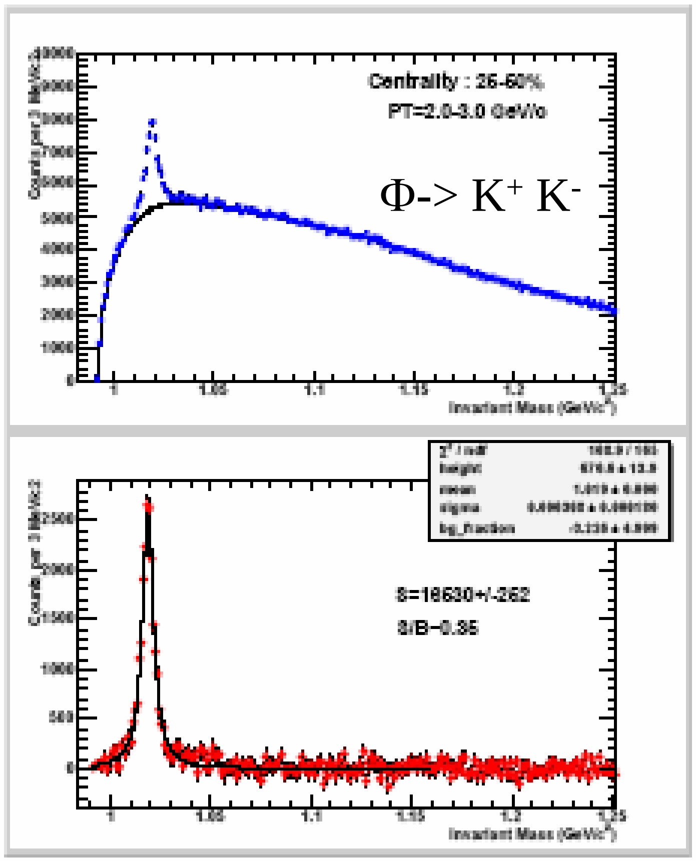
PHOBOS PID Capabilities



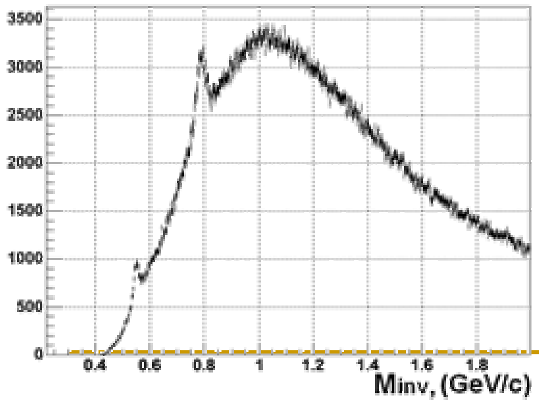
Particle ID from low to high p_T

Neutral particles can be reconstructed through their decay product

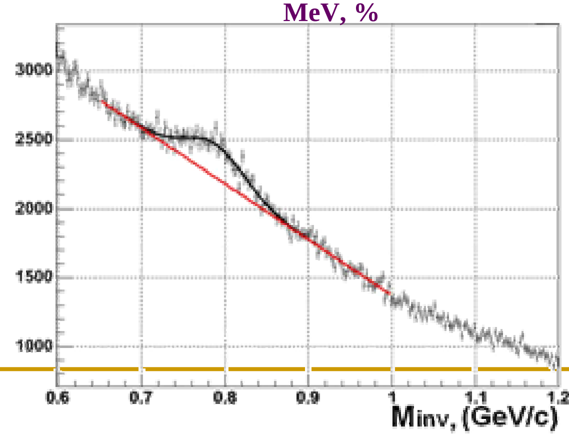




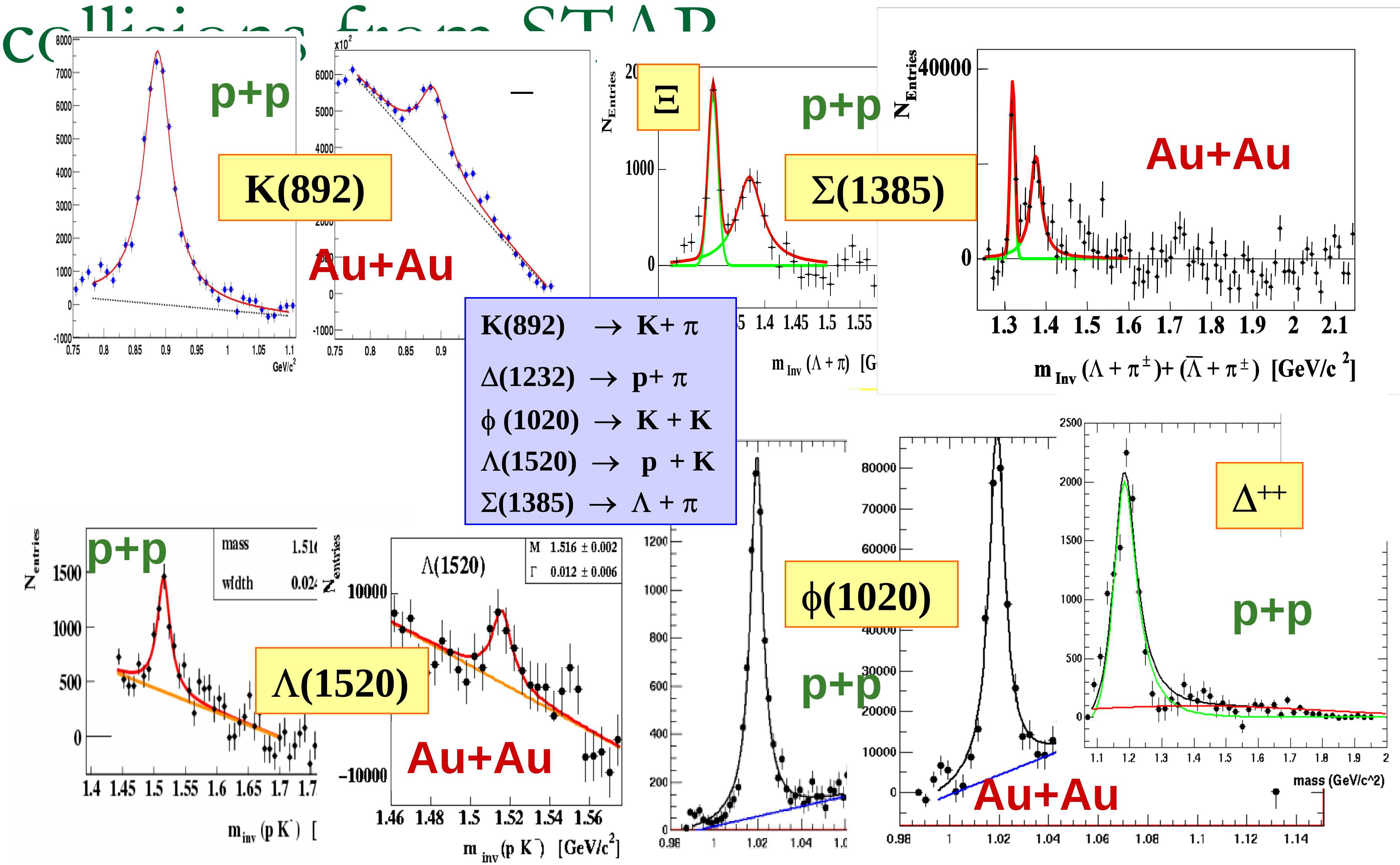
$\pi^+ \pi^- \pi^o$ decay channel in pp:
ω: m = 782.7 ± 0.1 MeV, BR = 89.1 ± 0.7%
η : m = 547.8 ± 0.1 MeV, BR = 22.6 ± 0.4%



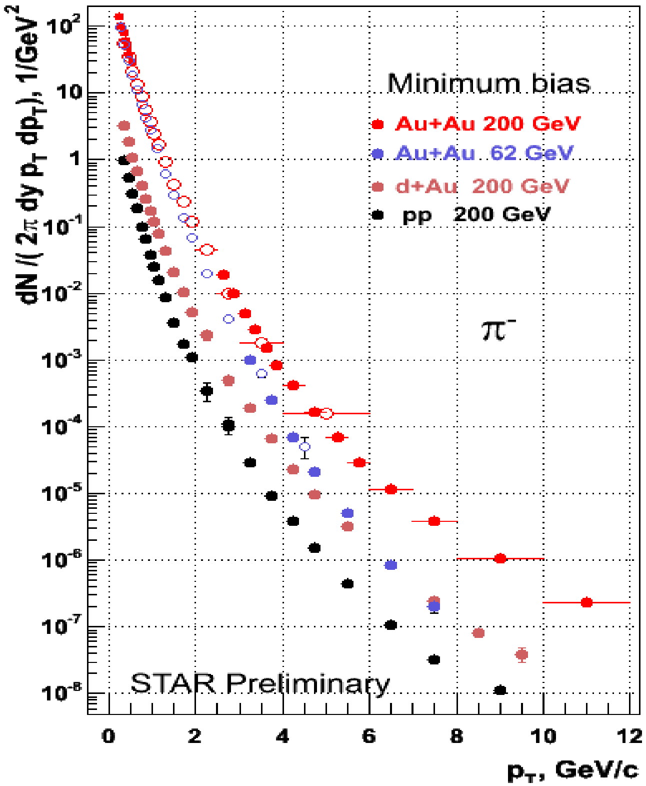
$\pi^o \gamma$ decay channel in pp:
ω - meson: m = 782.65 ± 0.1289 MeV, %

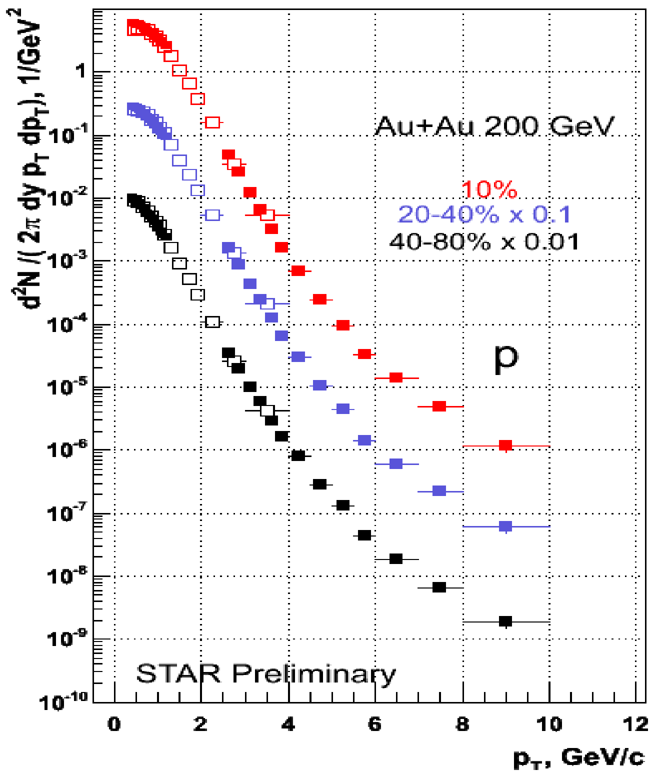


Resonance Signals in p+p and Au+Au collisions from STAR



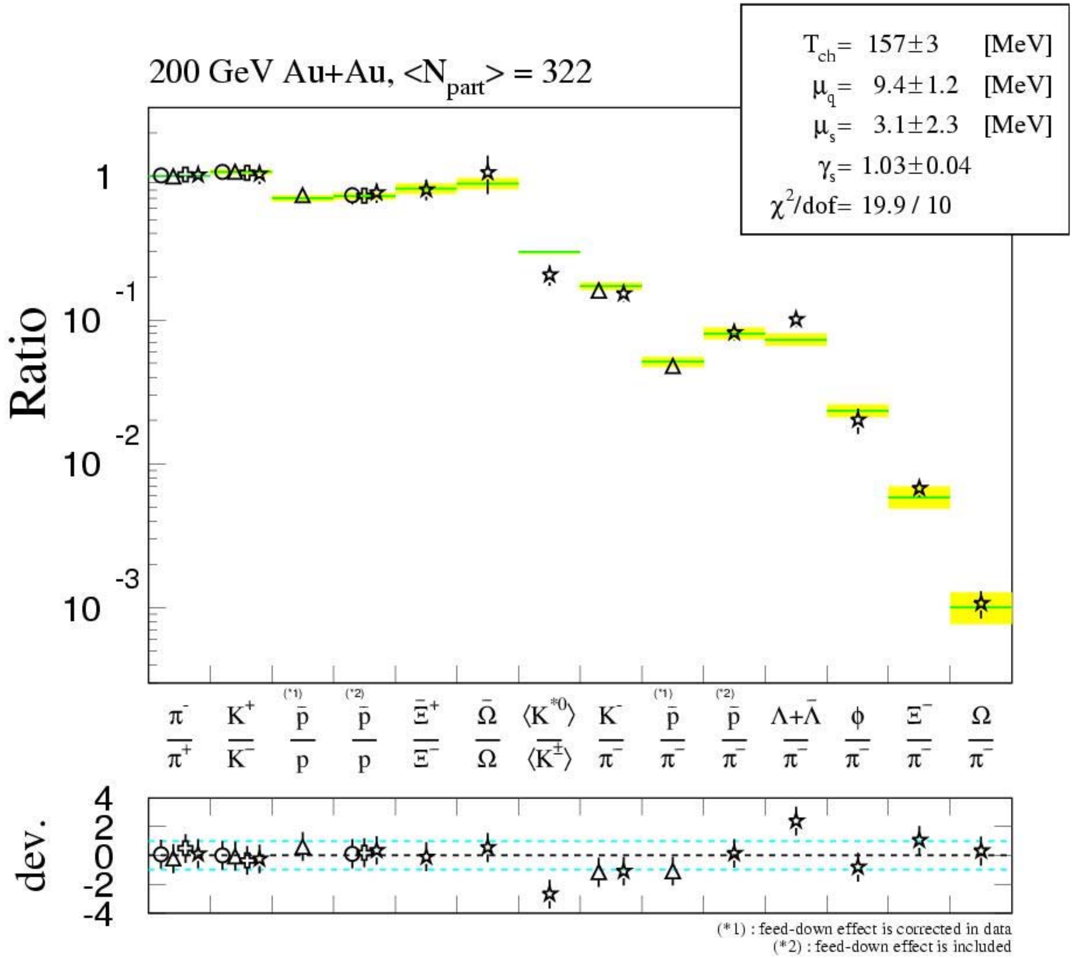
Measure particle spectra





- Corrections
  - Acceptance, efficiency (maybe multiplicity dependent)
  - PID purity
  - Feed-down from decays
  -

Statistical model fits: T_{ch} and μ_b

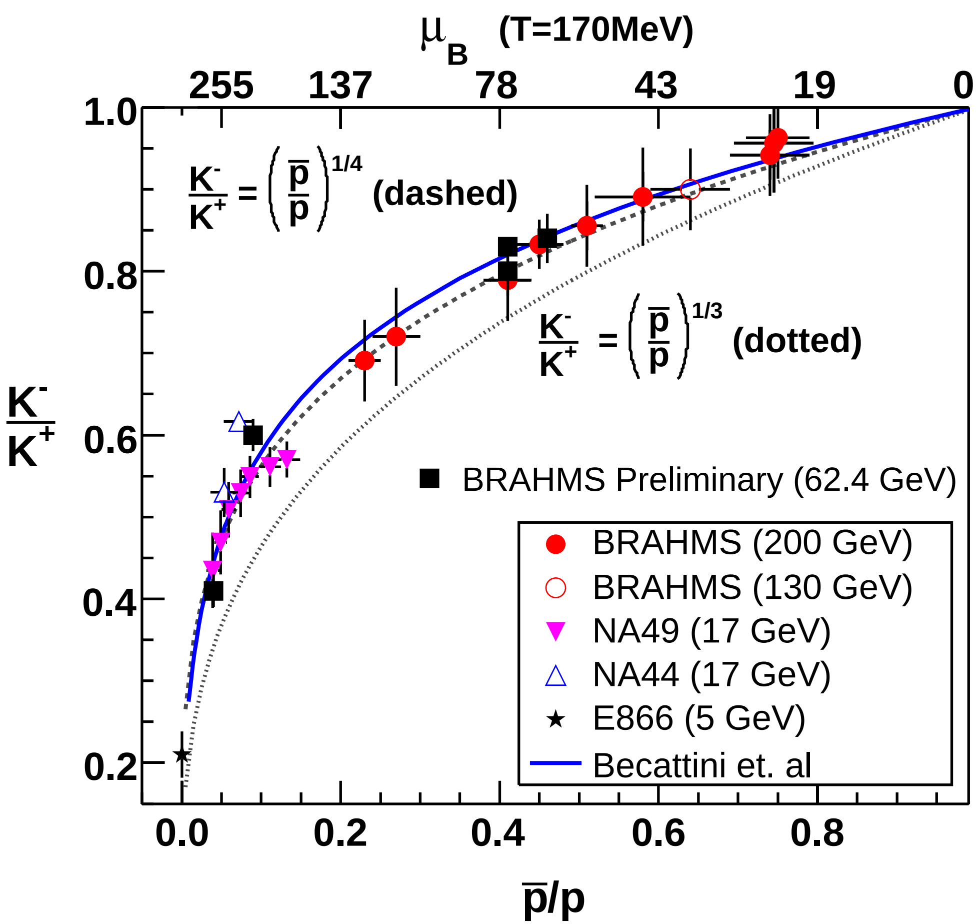


Look like the system has established thermal equilibrium at some point in its evolution (we don't know when from this type of analysis, but we have other handles)

The chemical abundances correspond to T_{ch} ~ 157+/- 3 MeV , μ_B ~ 30 MeV

Short lived resonances fall off the fits

The baryon chemical potential



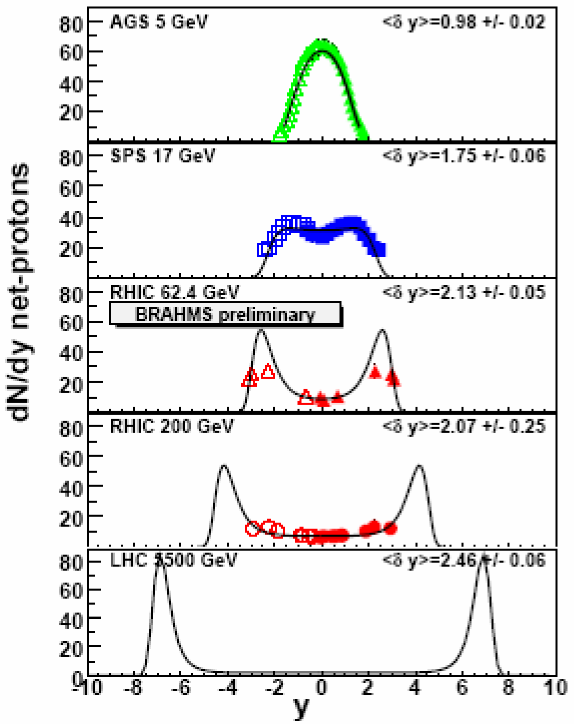
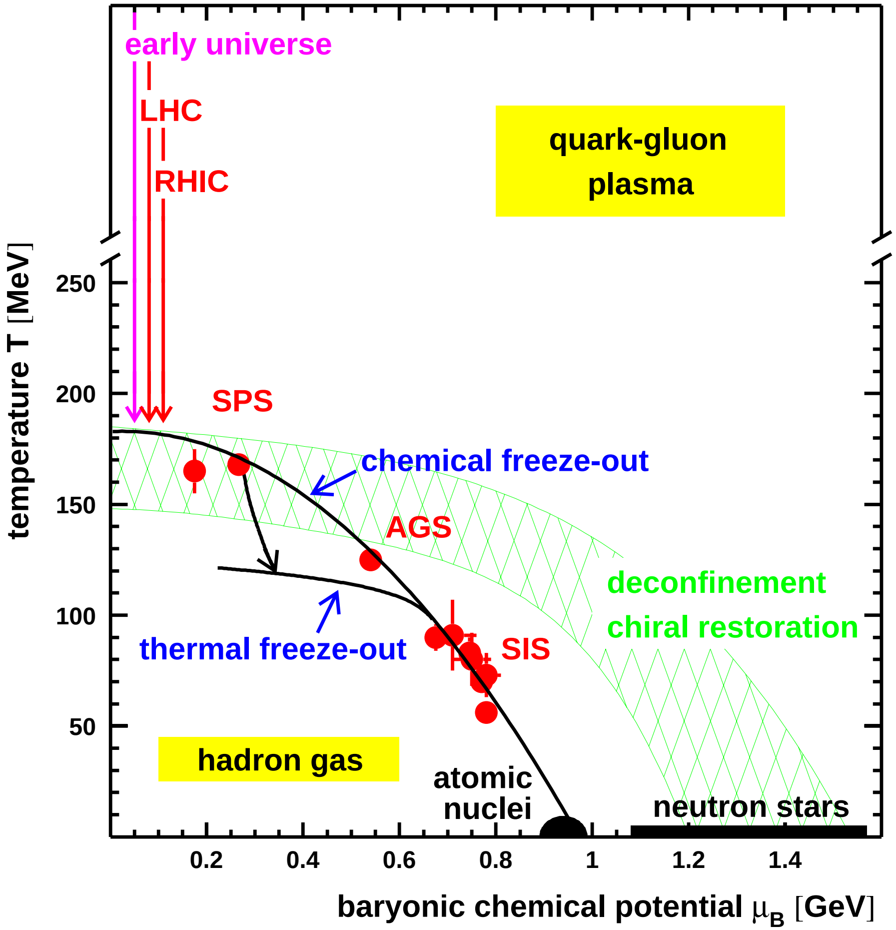


Fig. 3. $\frac{dN}{dy}$ of protons from AGS[6], SPS[9] and RHIC[5]. Errors include systematic errors. Bottom panel shows extrapolation to LHC.

Where are we on the phase diagram ?



PBM et al, nucl-th/0304013

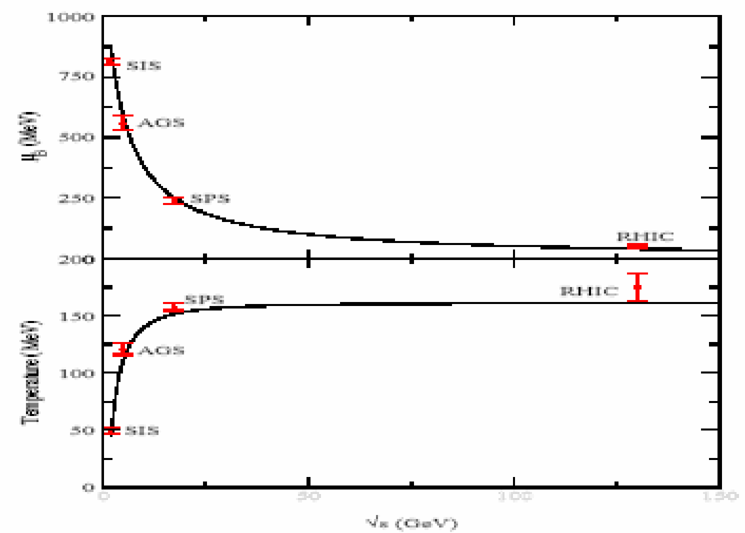


Fig. 31. Behavior of the freeze–out baryon chemical potential μ_B (upper curve) and the temperature T (lower curve) as a function of energy from Ref. (53). The temperature T as a function of beam energy is determined from the unified freeze–out conditions of fixed energy/particle.

What is the order of the phase transition ?

1st order

- Is there a phase transition at RHIC and LHC ?
- From lattice – it is a cross-over
- Then QGP or not is not a “yes” or “no” answer
- Smooth change in thermodynamic observables
- Can we find the critical point ?
  - Then we’ll have dramatic fluctuations in <p_T> and baryon number
- Data on fluctuations at SPS and RHIC – very similar results and no dramatic signals. Are we on the same side of the critical point ?

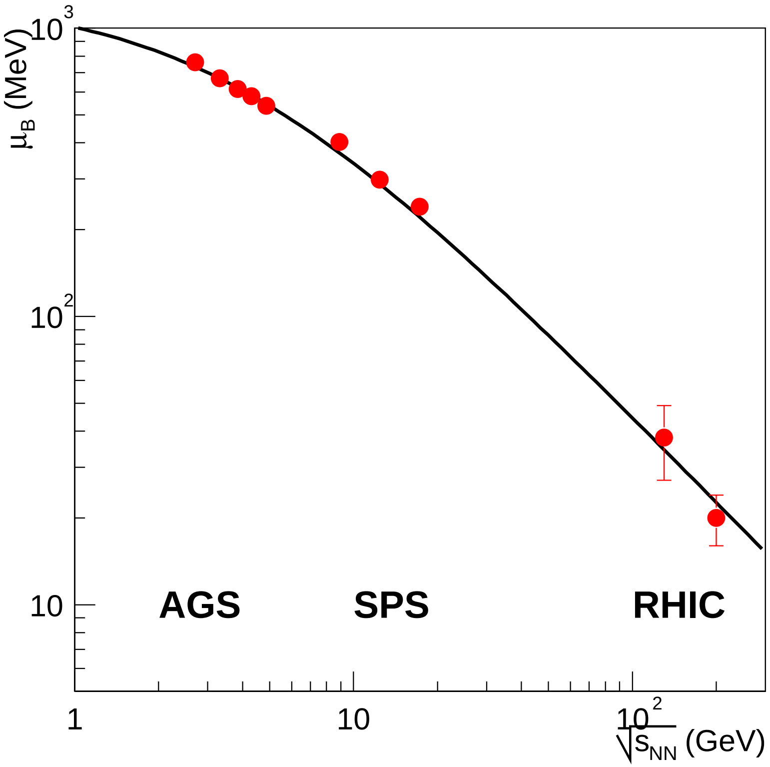
- While T_c is rather well established, there is a big uncertainty in μ_b

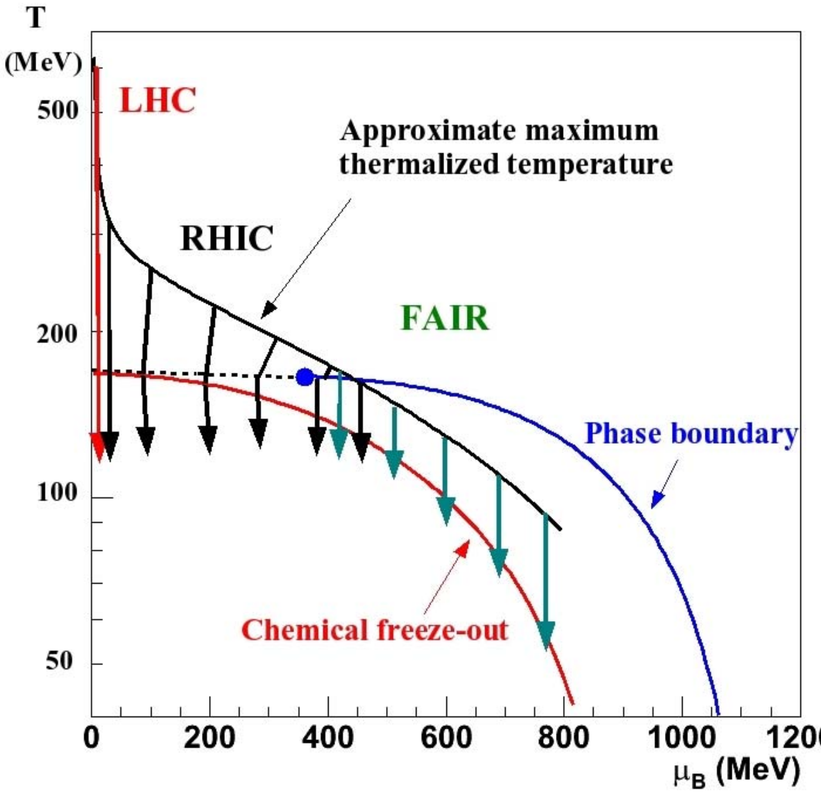
$\mu_b^{endpoint}/ T_c$ ~ 1 (Gavai, Gupta), ~ 2 (Fodor,Katz), ~ 3 (Ejiri et al)

$\mu_b^{freezout}$ 450 MeV (AGS) --→ 30 MeV (RHIC)

$\mu_b^{freezout}$ ~ T_c corresponds to sqrt(s) = 25 GeV

Can we find the critical point ?





- Large range of μ_B still unexplored : no data in the range μ_B = 70 -240 MeV
- You can run RHIC at low energies (with some work on the machine which seems feasible). The cover μ_B = 30-500 MeV ($\sqrt{s_{NN}}$ from 5 GeV to 200 GeV)
- The baryon chemical potential coverage at FAIR will be approximately 400-800 MeV.

Initial conditions

- Two pieces of information needed to establish the initial conditions:
  - the critical energy density ε_c required for deconfinement
  - the equation of state (EoS) of strongly interacting matter
- Lattice QCD determines both ε_c and EoS

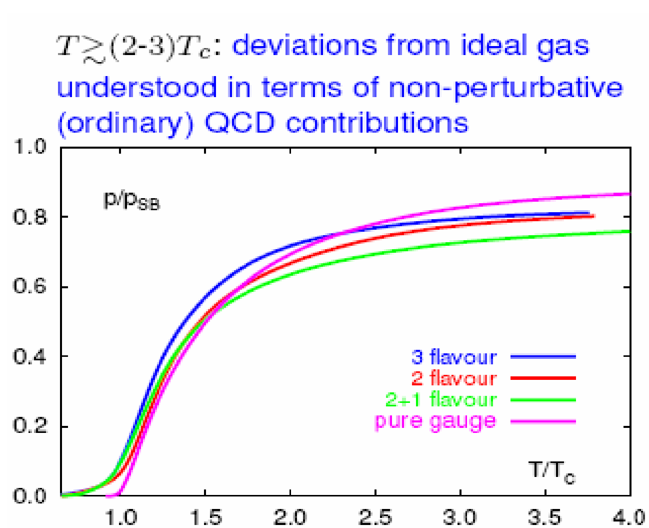
Lattice QCD – QGP phase transition

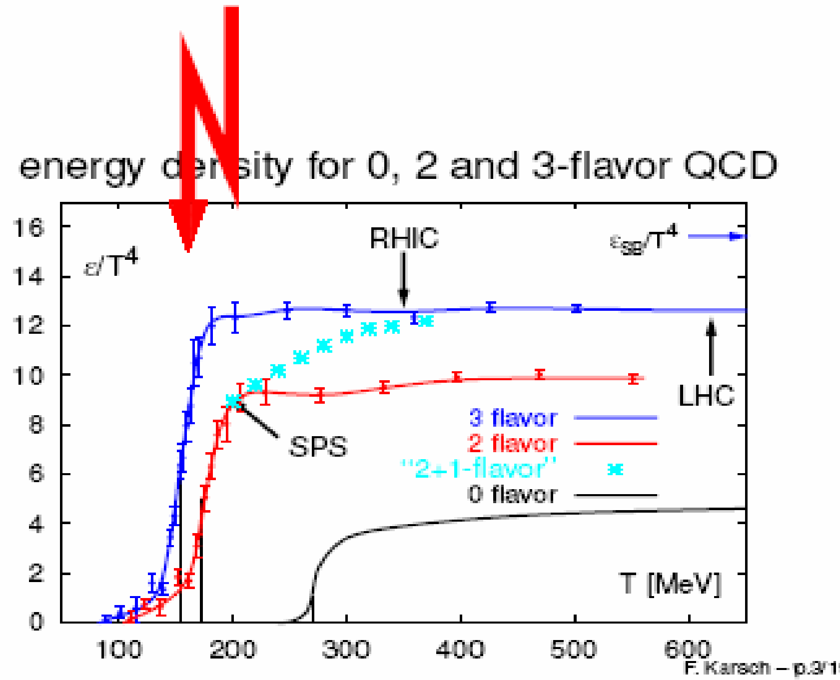
$\varepsilon_{SB} = n_f\, \pi^2 /30\; T^4$

$T_C \sim 155\text{-}175$ MeV $\quad \varepsilon_C \sim 0.3\text{-}1.0$ GeV/fm^3

n_f in hadron gas: 3 (π^+, π^-, π^0)

$$\varepsilon_{SB} = \begin{array}{l} \{2_f \cdot 2_s \cdot 2_q \cdot 3_c \frac{7}{8} + 2_s \cdot 8_c\} \frac{\pi^2}{30} T^4 = 37\, \frac{\pi^2}{30} T^4 \\ \{3_f \cdot 2_s \cdot 2_q \cdot 3_c \frac{7}{8} + 2_s \cdot 8_c\} \frac{\pi^2}{30} T^4 = 47.5\, \frac{\pi^2}{30} T^4 \end{array}$$



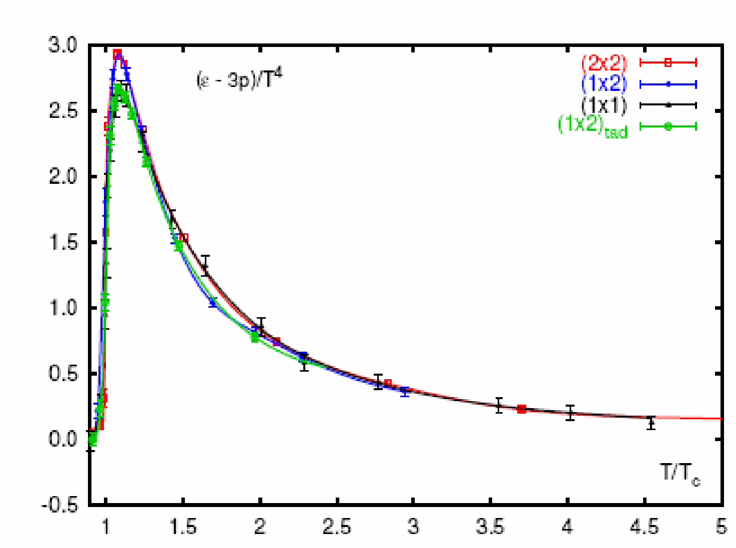


$T \lesssim 2T_c$: strong deviations from ideal gas
large screening masses,
remnants of confinement

F. Karsch – p.4/19

L-QCD : EoS

EoS for pure glue: strong deviations from ideal gas up to 2 T_c



- L-QCD – the only theory that can compute the EoS from first principles
- But, l-QCD lacks dinamical effects of the finite nuclear collision system.
- Many of the global observables are strongly influenced by the dynamics of the collisions.
- Microscopic (for the initial state) and macroscopic (hydrodynamics) transport models describe the collective dynamics: EoS is used as an input, local thermal equilibrium is assumed at all stages, system evolution is computed => results compared to data

Statistical model in pp collisions

First proposed by Rolf Hagedorn in order to describe the exponential shape of the mt-spectra of produced particles in p–p collisions. Hagedorn also pointed out phenomenologically the importance of the canonical treatment of the conservation laws for rarely produced particles.

Recently a complete analysis of hadron yields in p–p as well

as in $\bar{p}$–p, e+e−, π–p and in K–p collisions at several center-of-mass energies has been done (refs) . This detailed analysis has shown that particle abundances in elementary collisions can be also described by a statistical ensemble with maximized entropy. In fact, measured yields are consistent with the model assuming the existence of equilibrated fireballs at a temperature T ≈160-180 MeV. However, the agreement with data requires a strangeness under-saturation factor γ_s ~0.51

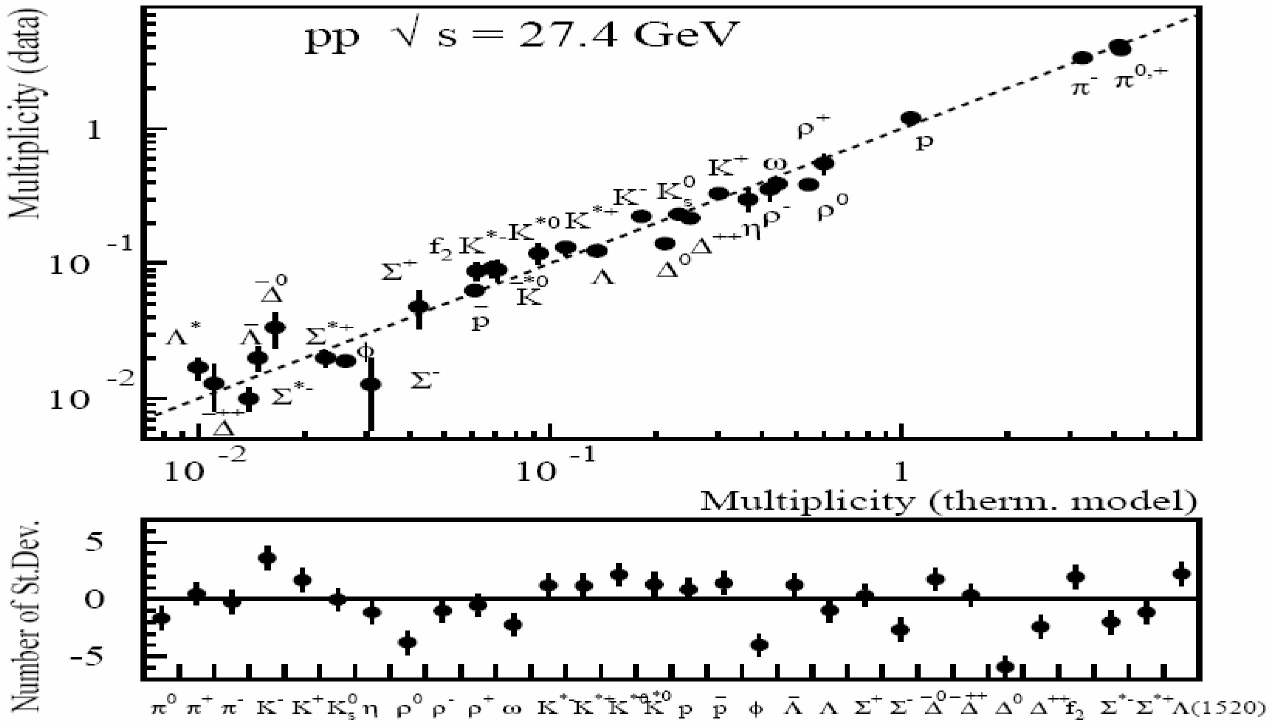
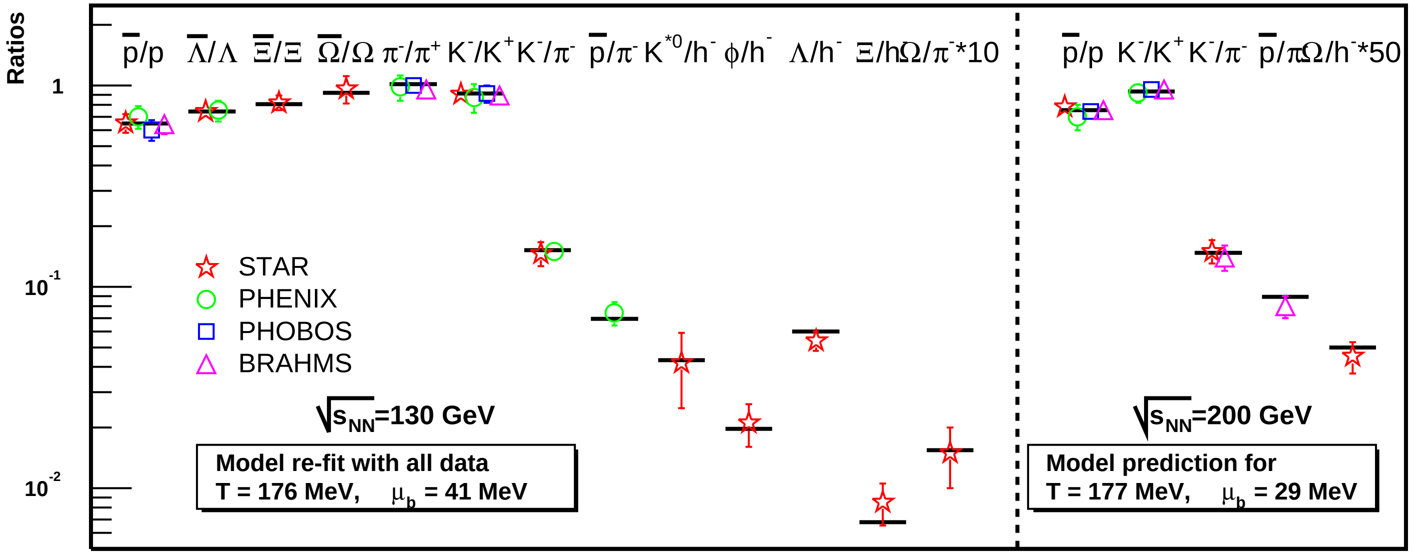


Fig. 19. A comparison[133] of the p–p multiplicity data at $\sqrt{s} = 27.4$ with the statistical model that accounts for the exact conservation of baryon number, electric charge, strangeness and includes the strangeness undersaturation factor $\gamma_s \simeq 0.51$.


Ratios
1
10^-1
10^-2
p̄/p Λ̄/Λ Ξ̄/Ξ Ω̄/Ω π-/π+ K-/K+ K-/π- p̄/π- K*0/h- φ/h- Λ/h- Ξ/h- Ω/π-*10
p̄/p K-/K+ K-/π- p̄/π- Ω/h-*50
STAR
PHENIX
PHOBOS
BRAHMS
√sNN=130 GeV
Model re-fit with all data
T = 176 MeV, μb = 41 MeV
√sNN=200 GeV
Model prediction for
T = 177 MeV, μb = 29 MeV