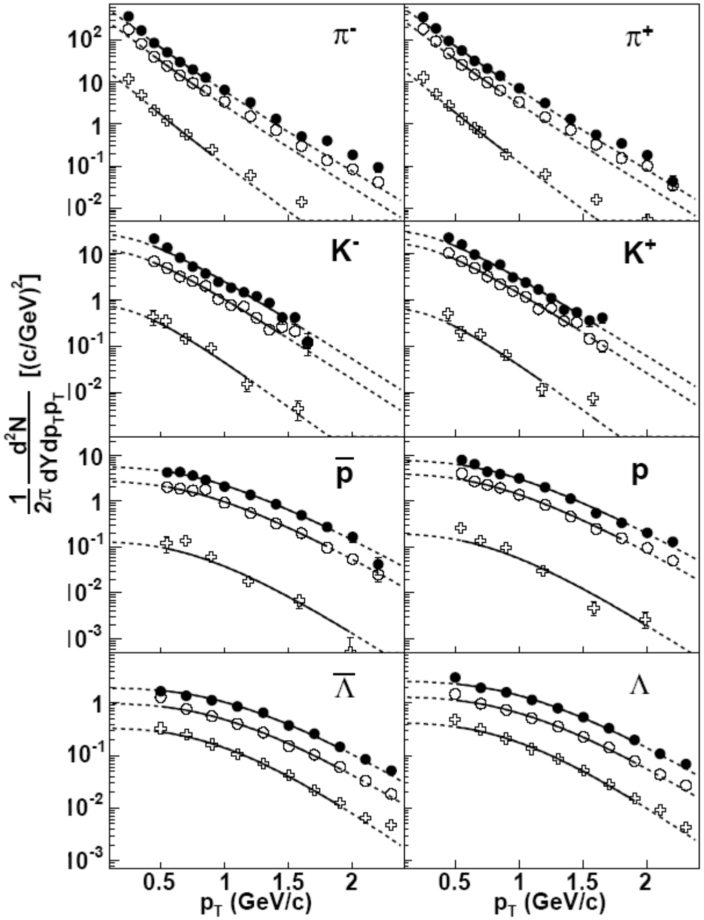
Lecture 12: Hydrodynamics in heavy ion collisions. Elliptic flow

- Last lecture we learned:
  - Particle spectral shapes in thermal model (static medium) are exponential in m_T with common slope for all particles. “m_T – scaling”
  - The slope is related to the temperature at decoupling (freeze-out)
  - In an expanding medium (AA collisions), the slopes are no longer constant with mass
    - mass ordering at low m_T
    - Common slope at high m_T

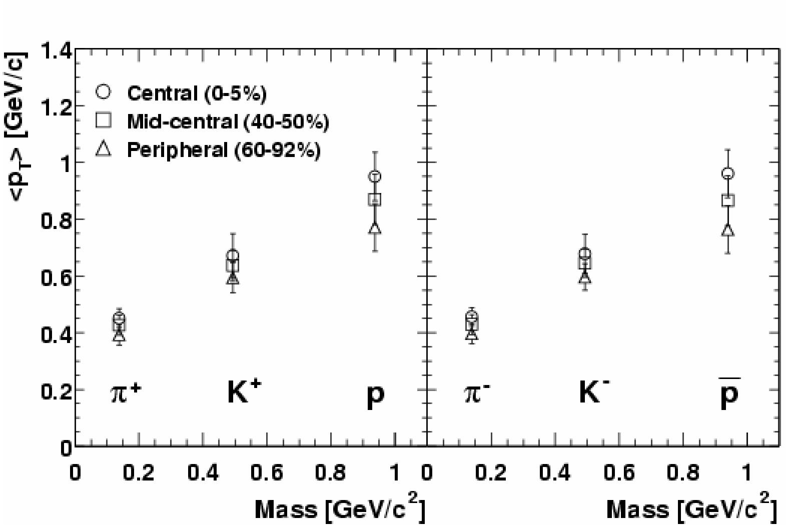
$$T_{eff} = T_{fo}\sqrt{\frac{1+\beta}{1-\beta}}$$

“blast wave” fits to spectra

Retiere and Lisa – nucl-th/0312024



PHENIX - Phys. Rev. C 69, 034909 (2004)



- Hydrodynamics inspired parameterization
- Obtain from fit:
  - Flow velocity
  - Freeze-out temperature

Today:

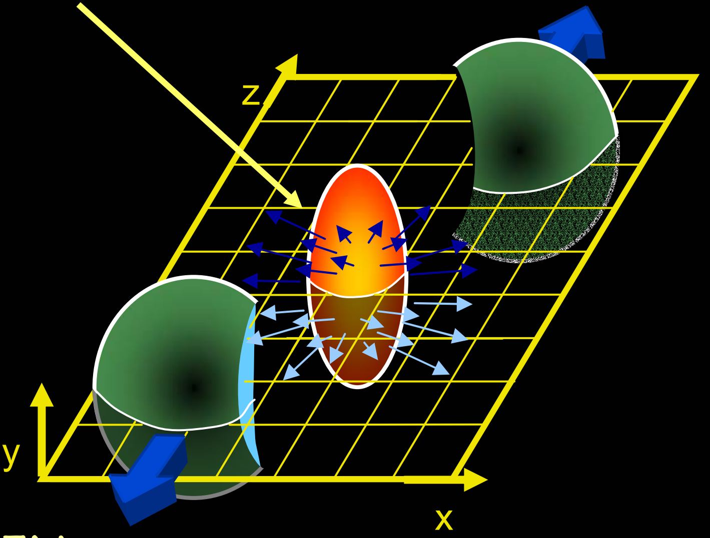
- Introduce a new observable (elliptic flow) sensitive to the early stage of the collisions
- More about how hydrodynamics works and what we learn from it

The Geometry of a Heavy Ion Collision

N_{part} Participants that undergo

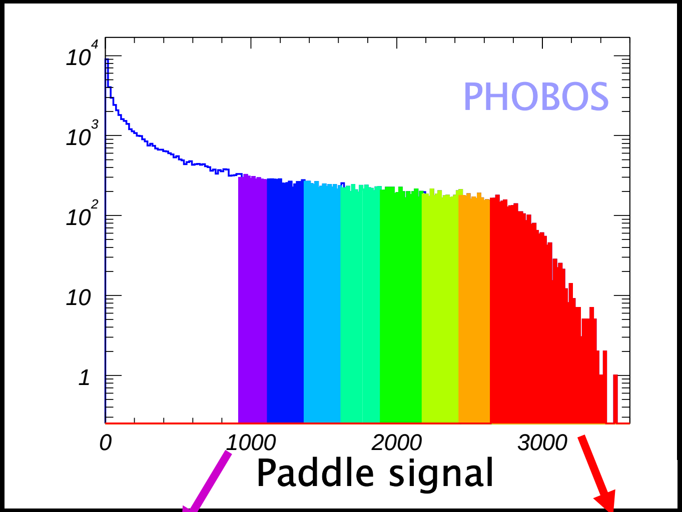
N_{coll} Collisions

$A-0.5N_{part}$ Spectators

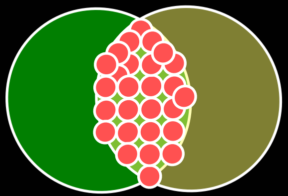


$A-0.5N_{part}$ Spectators

…and we can measure this!

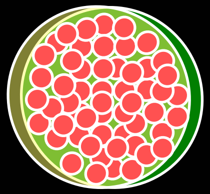


<u>Peripheral Collision</u>:



→ Small number of participating nucleons

<u>Central Collision</u>

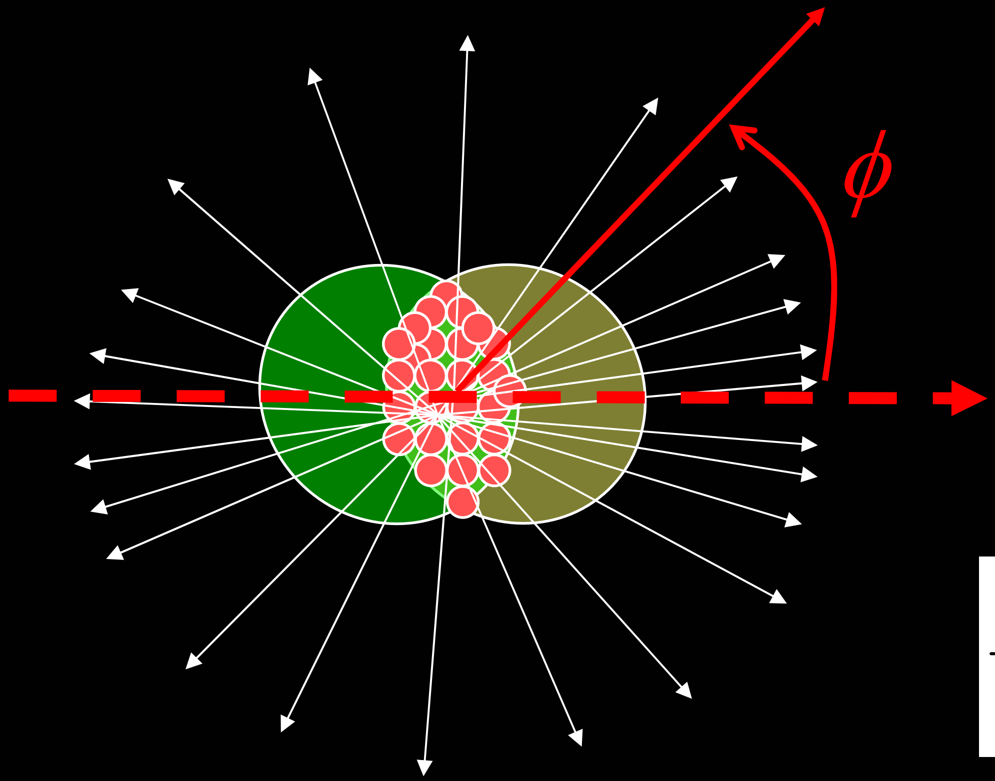


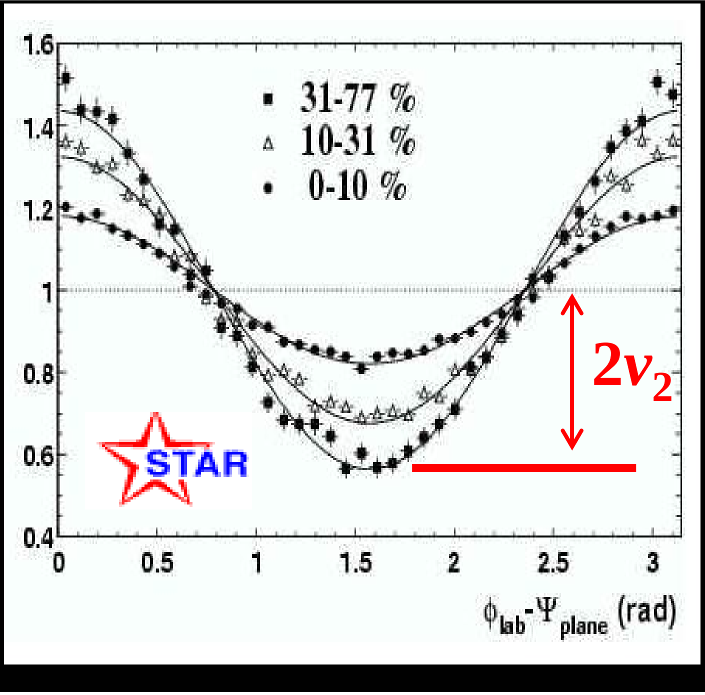
→ Large N_{part}

→ We can classify collisions according to centrality.

State of Matter appears strongly interacting

(Similar to a "fluid")





$$\frac{dN}{d\phi} \propto 1 + 2v_2(p_T)\cos(2\phi) + \ldots$$

"elliptic flow"

→ **Experiment finds a clear v_2 signal**

→ **If system was freely streaming the spatial anisotropy would be lost**

Basics of Hydrodynamics

Hydrodynamic Equations

$$\partial_\mu T^{\mu\nu} = 0,$$ Energy-momentum conservation

$$\partial_\mu n_i^\mu = 0$$ Charge conservations (baryon, strangeness, etc…)

For perfect fluids (neglecting viscosity),

$$T^{\mu\nu} = (\underbrace{\boxed{e}}_{\text{Energy density}} + \underbrace{\boxed{P}}_{\text{Pressure}})u^\mu \underbrace{\boxed{u^\nu}}_{\text{4-velocity}} - P g^{\mu\nu}$$

Need equation of state (EoS)

$P(e, n_B)$

to close the system of eqs.
→ Hydro can be connected directly with lattice QCD

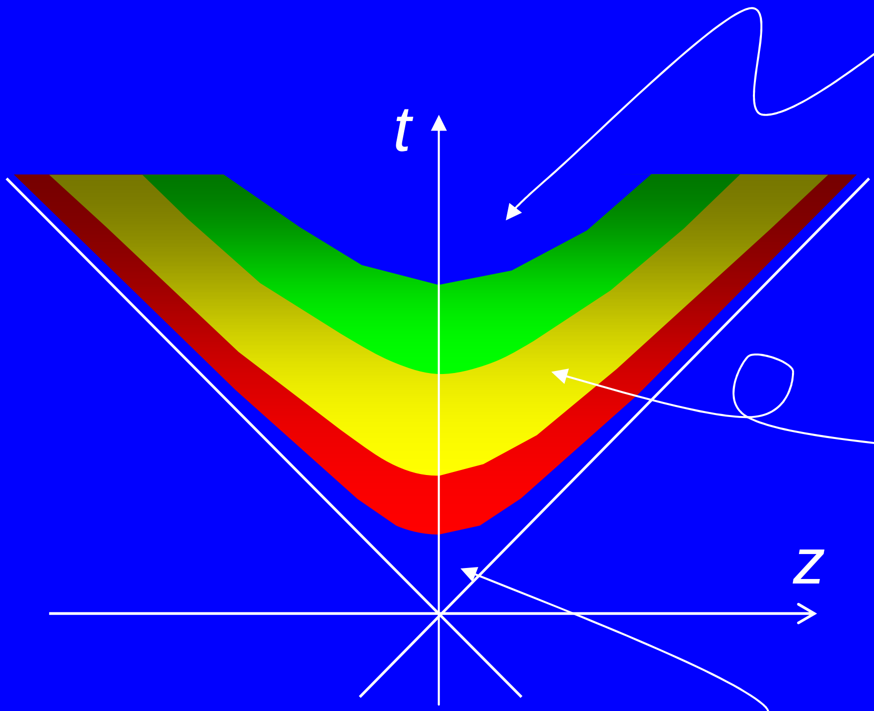
Within ideal hydrodynamics, pressure gradient *dP/dx* is the driving force of collective flow.

→ Collective flow is believed to reflect information about EoS!

→ Phenomenon which connects 1st principle with experiment

Caveat: Thermalization, $\lambda \ll$ (typical system size)

Inputs to Hydrodynamics



Final stage:
Free streaming particles
→ Need decoupling prescription

Intermediate stage:
Hydrodynamics can be valid if thermalization is achieved.
→ Need EoS

Initial stage:
Particle production and pre-thermalization beyond hydrodynamics
→Instead, initial conditions for hydro simulations

Need modeling
(1) EoS, (2) Initial cond., and (3) Decoupling

Initial conditions

- Hydro requires thermal equilibrium (at least locally)
- Thus, the initial thermalization stage in a heavy ion collision lies outside the domain of applicability of the hydrodynamic approach and must be replaced by initial conditions for the hydrodynamic evolution.
- Different approaches explored:
  - treat the two colliding nuclei as two interpenetrating cold fluids feeding a third hot fluid in the reaction center (“three-fluid dynamics”). This requires modelling the source and loss terms describing the exchange of energy, momentum and baryon number among the fluids.
  - microscopic transport models: (parton cascades) VNI, VNI/BMS, MPC, AMPT estimate the initial energy and entropy distributions in the collision region before switching to a hydrodynamic evolution. However the thermalization mechanism is still poorly understood at a microscopic level

Initial conditions (continued)

- Assuming
  - isentropic expansion
  - Particle multiplicities in the final state (measured) define the entropy
- Need to go from: measured final multiplicity to initial distribution of energy density
- Use Glauber model to predict N_{part} and N_{coll} for a given impact parameter
- Density distribution of the nucleus

$$\rho_A(r) = \frac{\rho_0}{e^{(r-R_A)/\xi} + 1},$$

- Integrate along the path of each nucleon to get the nuclear thickness function and N_{part}, N_{coll}

Initial conditions

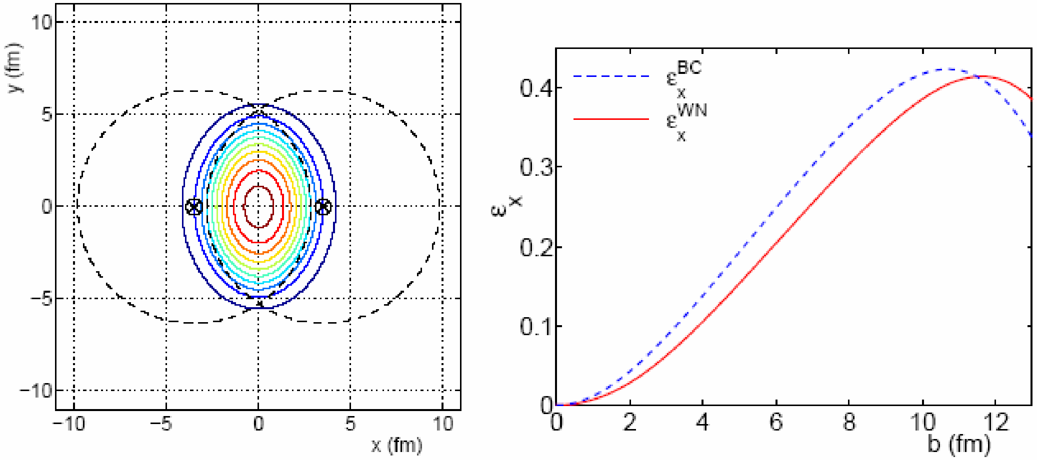
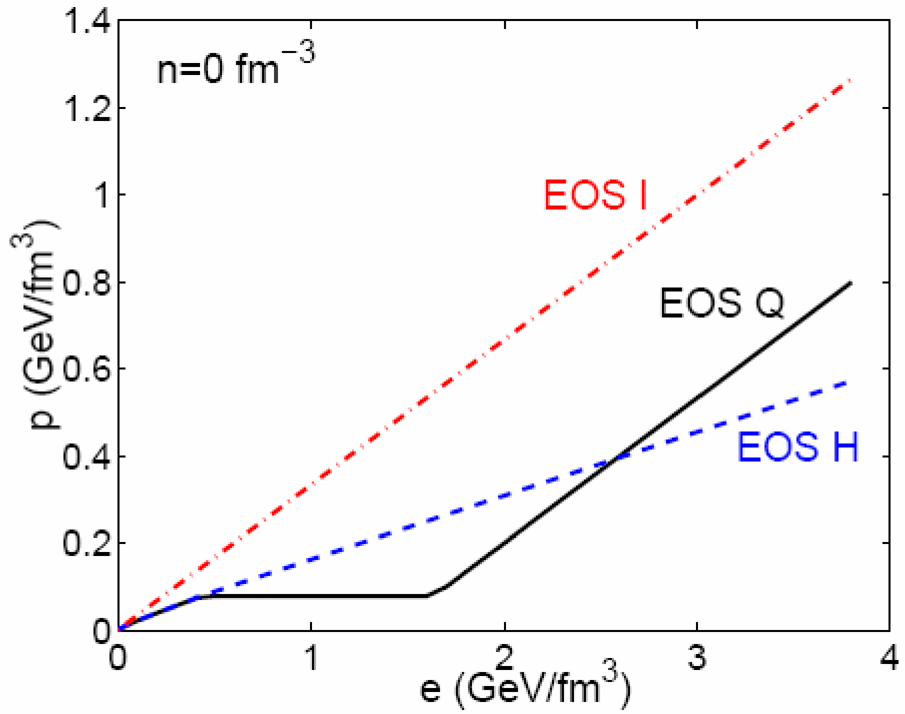


Fig. 3. Density of binary collisions in the transverse plane for a Au+Au collision with impact parameter $b = 7$ fm (left). Shown are contours of constant density together with the projection of the initial nuclei (dashed lines). The right plot shows the geometric eccentricity as a function of the impact parameter for the wounded nucleon and binary collision distributions.

- The initial entropy density and energy density is taken proportional to the a*N_{part} +b*N_{coll}

EoS

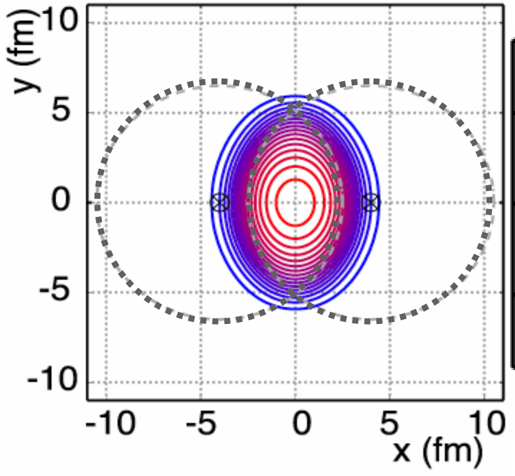
- EoS can either be modeled or extracted from lattice QCD calculations.
- Typically – modeled
  - low temperature regime: non-interacting hadron gas with (smallish) speed of sound $c_s^2 = \partial p/\partial e \approx 0.15$
  - Above the transition: free gas of massless quarks and gluons: $c_s^2 = \partial p/\partial e = 1/3$



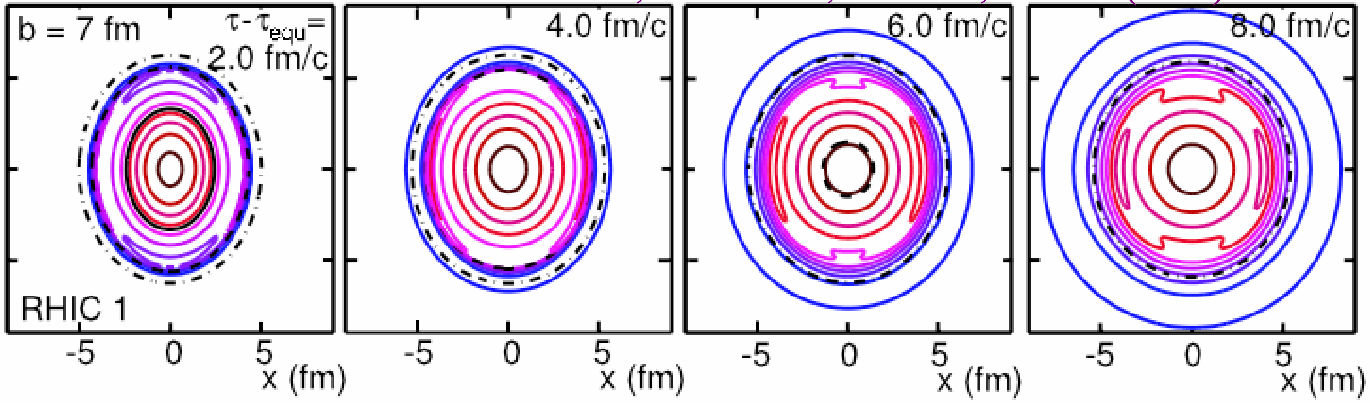
Decoupling

- hydrodynamic description begins to break down again once the transverse expansion becomes so rapid and the matter density so dilute that local thermal equilibrium can no longer be maintained.
- Rely on the fact that the entropy density, energy density, particle density and temperature profiles are directly related and all have similar shapes. Thus, decouple on a surface of constant temperature and convert the fluid cells to particles
- “Sudden freeze-out” goes from 0 mean free path to infinite mean free path – artificial
- Better method: a hybrid approach. After converting to particles – hand the output to a microscopic model that will allow for more re-scattering and a natural freeze-out when matter gets very dilute

Geometry converts to Momentum Space



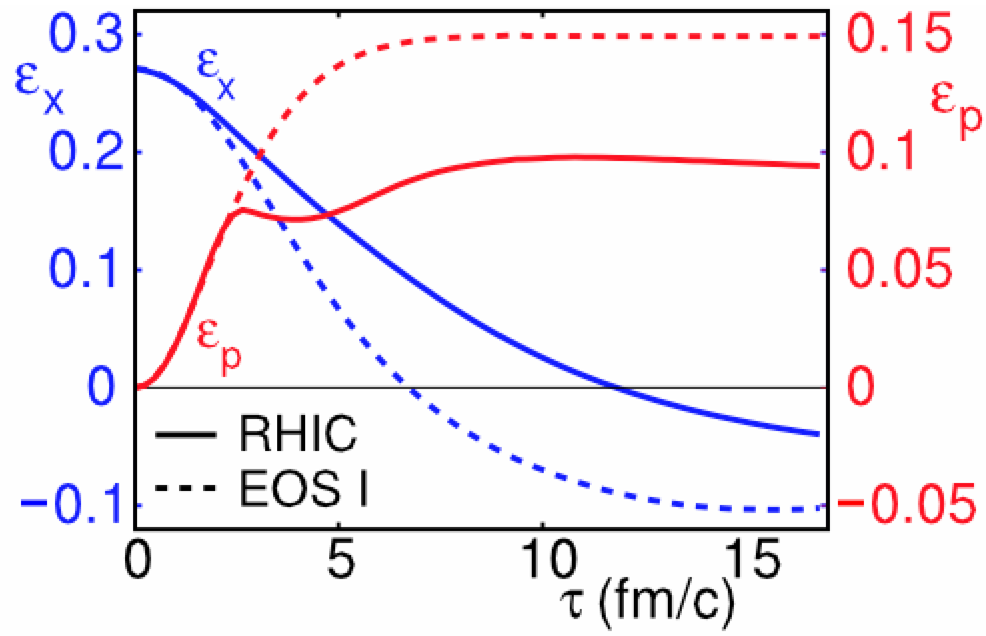
PFK, J. Sollfrank, U. Heinz, PRC 62 (2000) 054909



Time evolution of anisotropies

Coordinate space

$$\epsilon_x = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



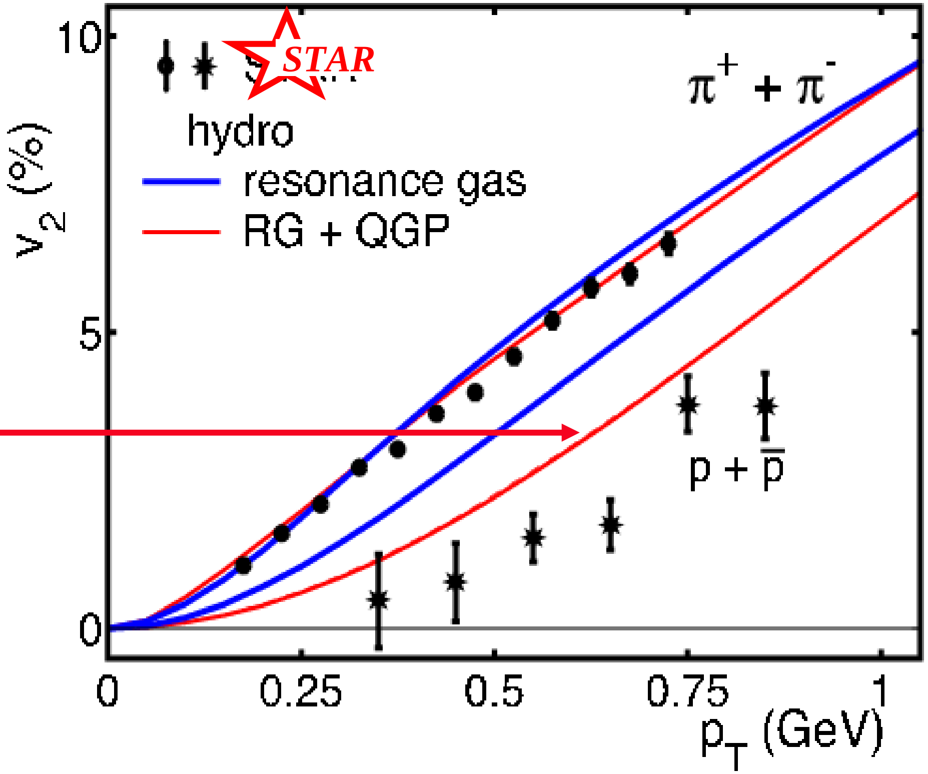
Momentum space

$$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

Collective effect probes equation of state

Hydrodynamics can reproduce magnitude of elliptic flow for π, p. BUT correct mass dependence requires *QGP EOS!!*

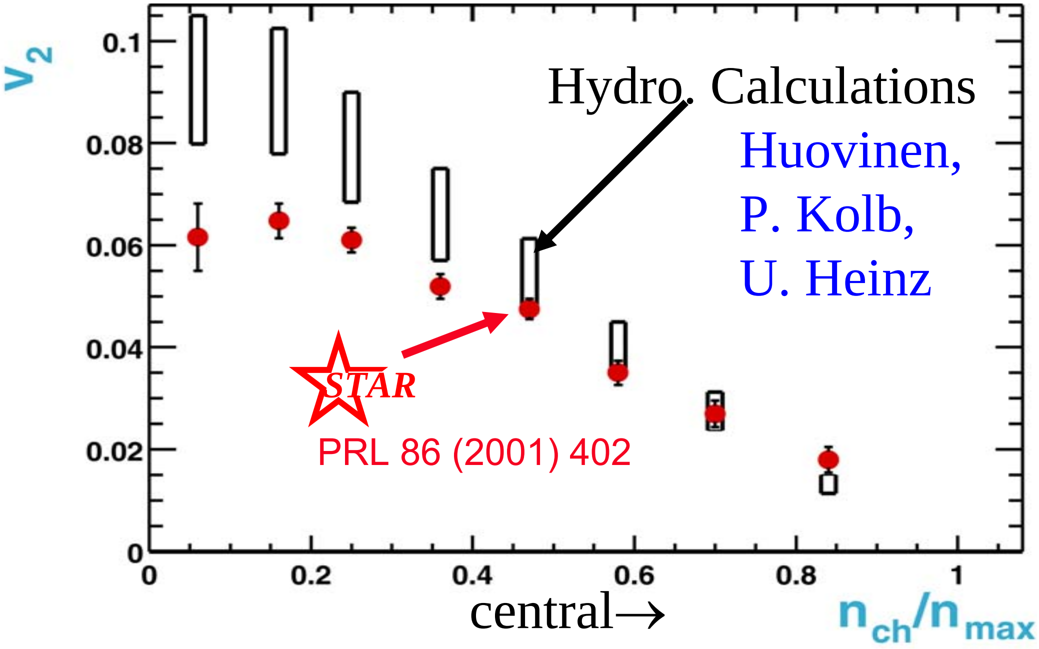
Kolb, et al



NB: these calculations have viscosity = 0 and 1s order phase transition.

We have concluded that medium behaves as an ideal liquid.

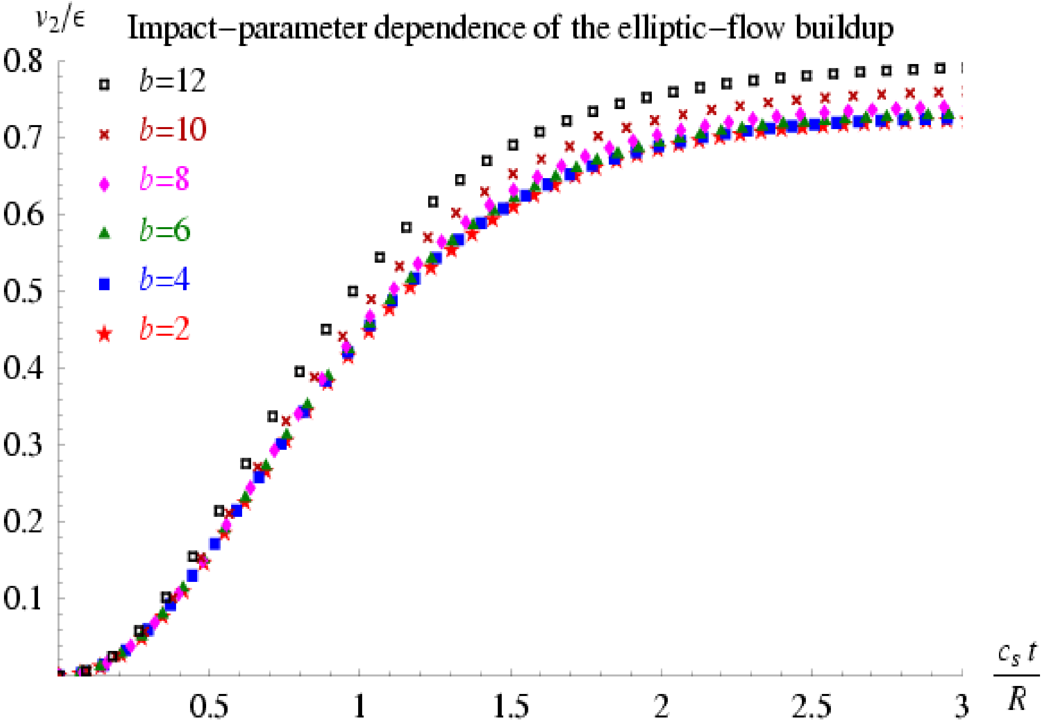
v_2 reproduced by hydrodynamics



- **see a large pressure buildup**
- **anisotropy → happens fast while system is deformed**
- **success of hydrodynamics→ early equilibration !**

  ~ 0.6 fm/c

Eccentricity scaling in hydrodynamics



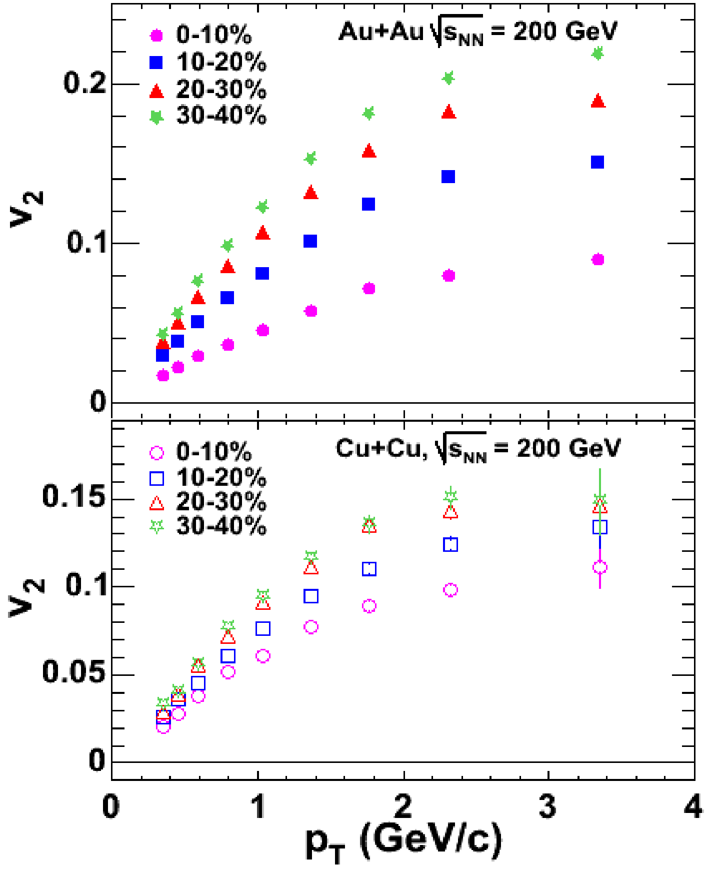
Bhalerao, Blaizot, Borghini, Ollitrault , nucl-th/0508009

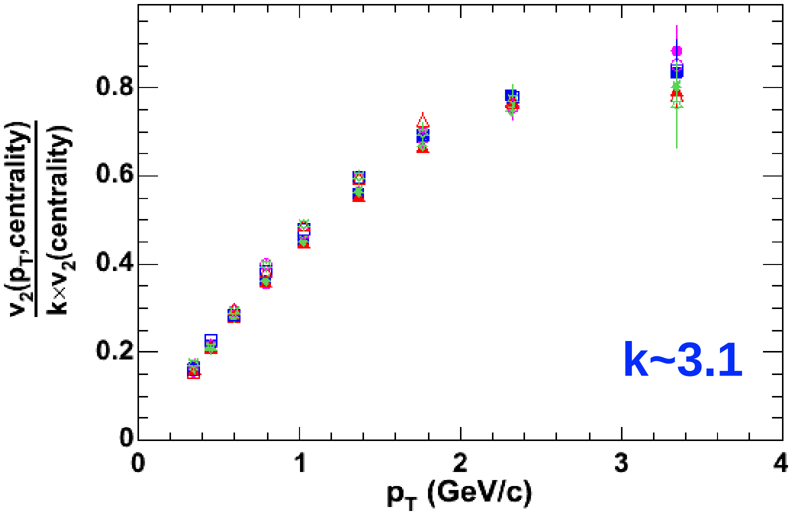
Eccentricity scaling observed in hydrodynamic model over a broad range of centralities

$$\frac{1}{R} = \sqrt{\frac{1}{\langle x^2\rangle} + \frac{1}{\langle y^2\rangle}}$$

R: measure of size of system

Eccentricity scaling in data





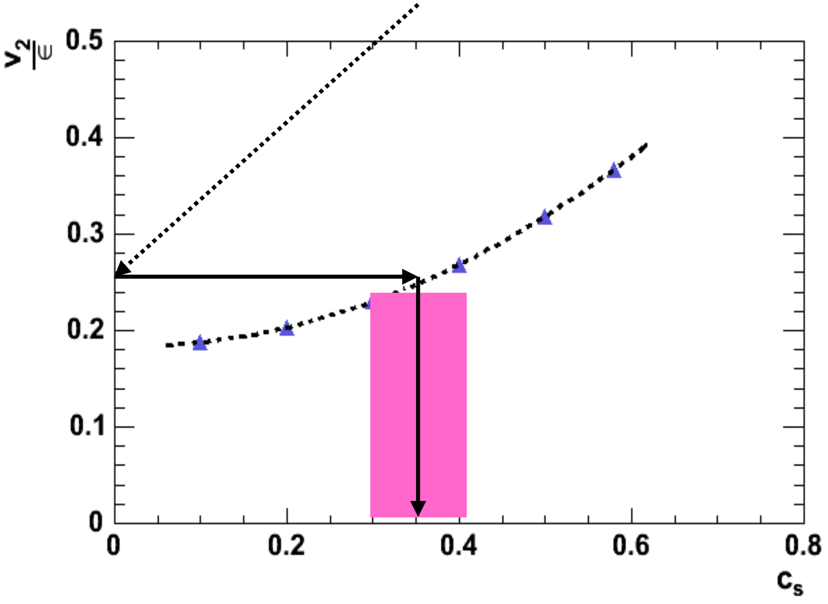
Cu has a smaller nuclear radius than Au, Hence, Cu+Cu collisions produce a smaller system than Au+Au for the same centrality

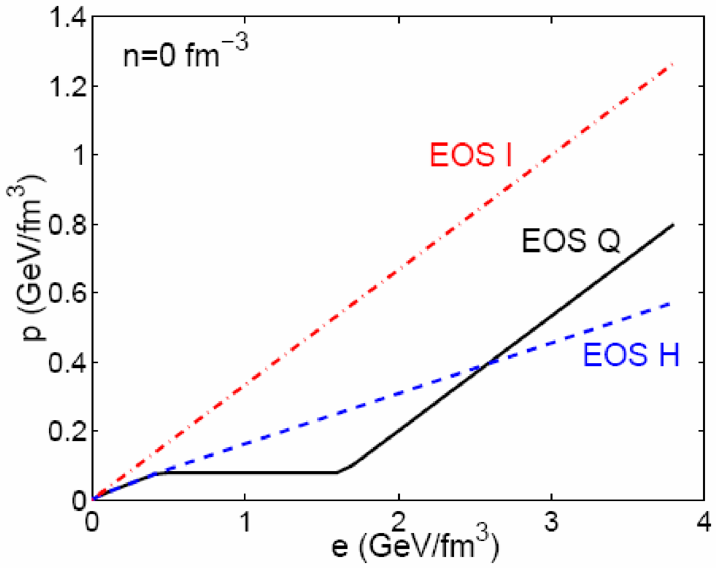
- v_2 scales with eccentricity
- for different centralities and different colliding systems
- Indicative of high degree of thermalization

Estimation of c_s

Equation of state (relation between pressure and energy density) can be written in terms of the speed of sound c_s → $p = c_s^2 \in$

v_2/ε for $\langle p_T \rangle \sim 0.45$ GeV/c (obtained from p_T spectra)





- **$c_s \sim 0.35 \pm 0.05$, ($c_s^2 \sim 0.12$), soft EOS**
- The matter does not spend a large amount of time in a mixed phase, indicating a weak first order phase transition or cross-over

Excitation function of v2: data vs theory

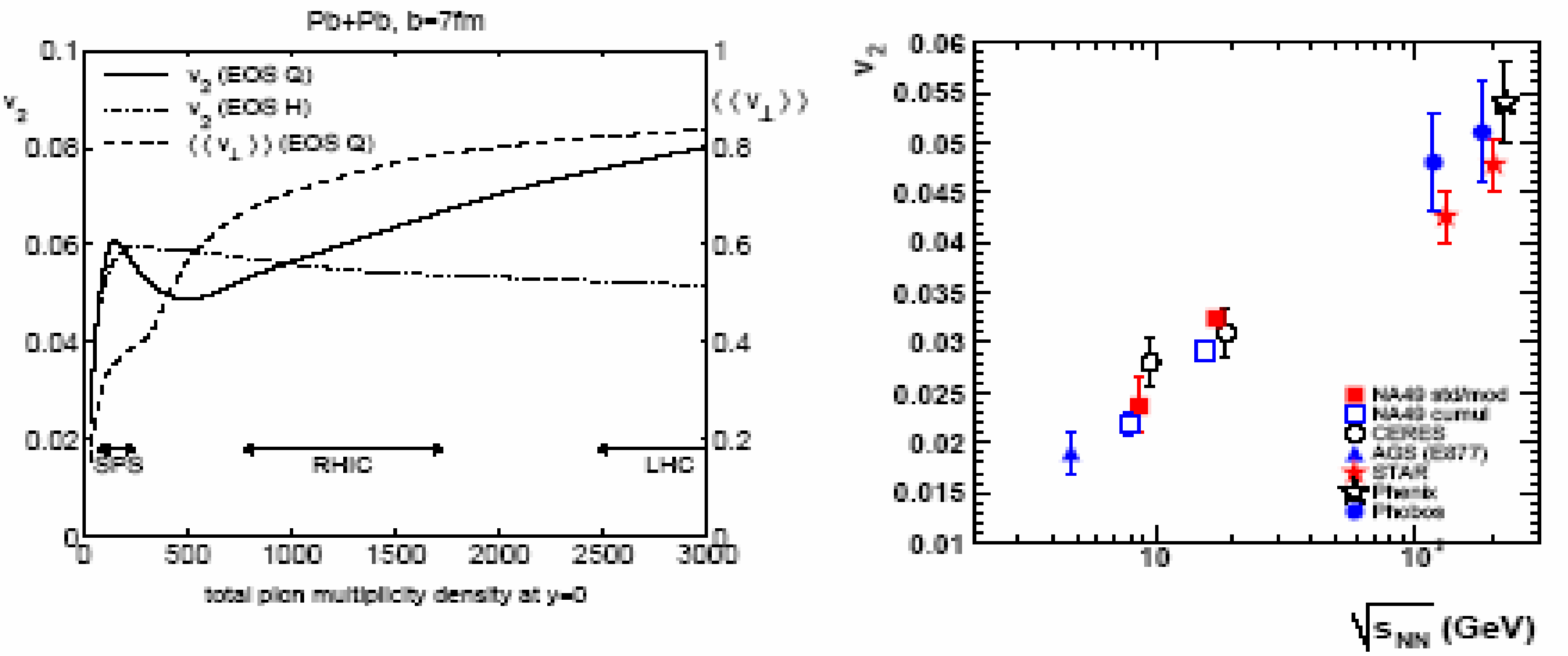


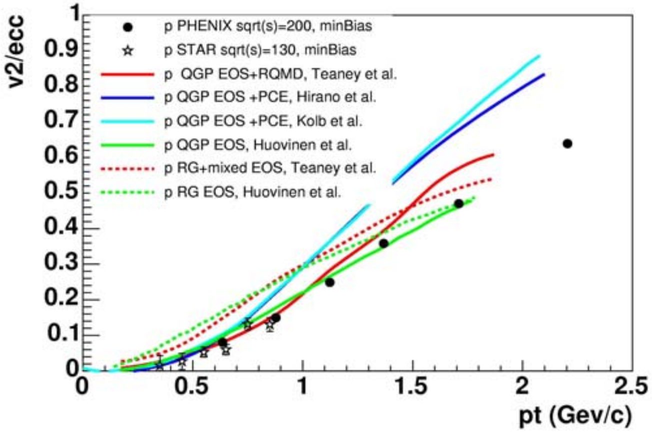
Fig. 24. Left: Excitation function of the elliptic (solid) and radial (dashed) flow for Pb+Pb or Au+Au collisions at $b = 7$ fm from a hydrodynamic calculation.[4] The collision energy is parametrized on the horizontal axis in terms of total particle multiplicity density dN/dy at this impact parameter. Right: A compilation of v_2 data vs. collision energy from midcentral (12–34% of the total cross section) Pb+Pb and Au+Au collisions.[122]

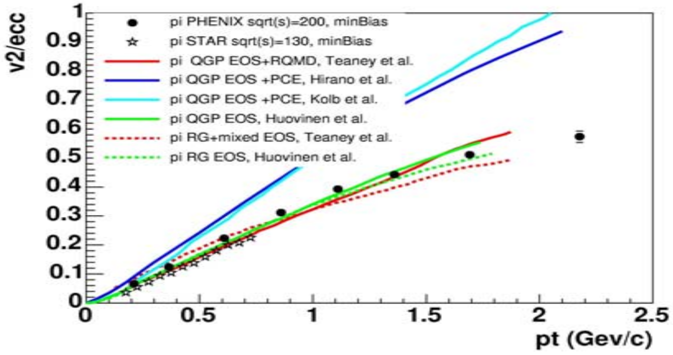
v_2 -AND- spectra

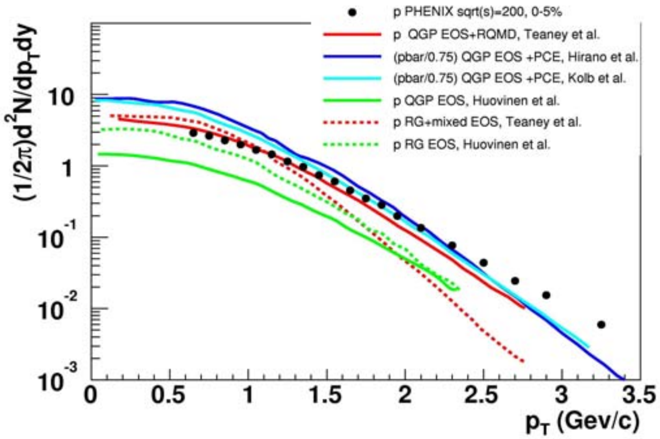
nucl-ex/0410003

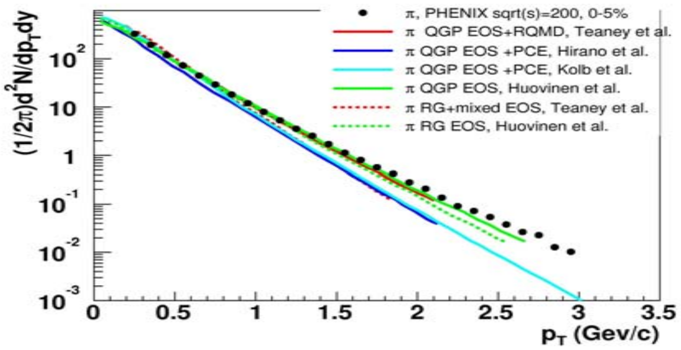
proton

pion









- **Not all hydro models describe all observables with the same set of parameters**
- **Need to model the decoupling stage microscopically to achieve agreement with spectra and v2 simultaneously**

Where else does hydro fail ?

- **In most early hydro calculations: boost invariance is assumed**
- **This simplifies a lot the hydro equations, because you don’t need to solve them in 3D , but rather 2D +time**
- **You pay the price that the calculations do not reproduce the v2 data a a function of rapidity**

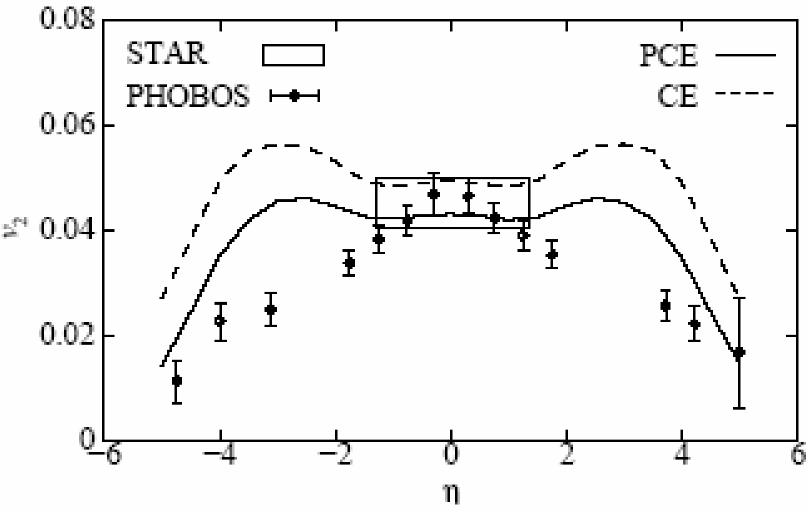


Fig. 25. p_T-integrated elliptic flow for minimum bias Au+Au collisions at $\sqrt{s} = 130\,A$ GeV as a function of pseudorapidity,[89] compared with data from PHOBOS and STAR.[145,124]

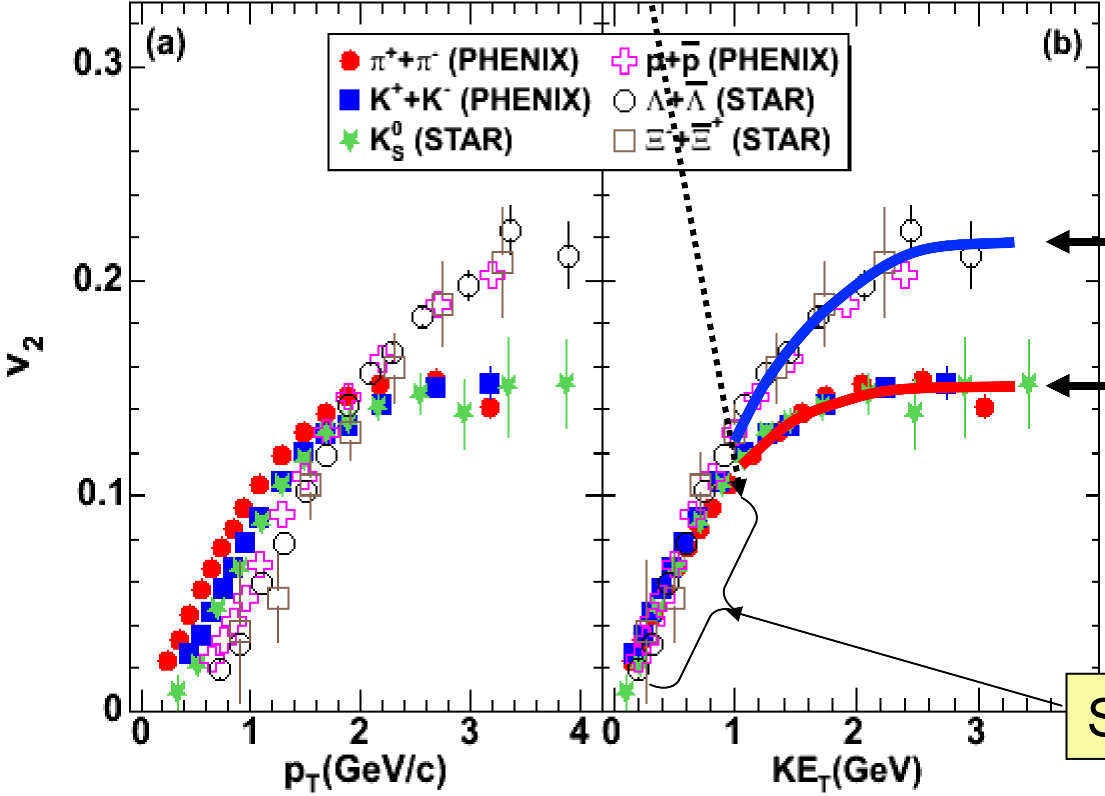
What have we learned from v2 data where hydro does work ?

I. Very rapid thermalization is required, to be able to build-up strong flow
II. Very small viscosity – because ideal hydro describes the data
III. The system is strongly coupled and behaves as a liquid
IV. 3D description is needed outside the mid-rapidity region
V. Microscopic description works best in describing the freeze-out conditions

VI. Next ask: what are the quanta that flow ? – in another lecture

Scaling v_2 with transverse kinetic energy

Scaling breaks



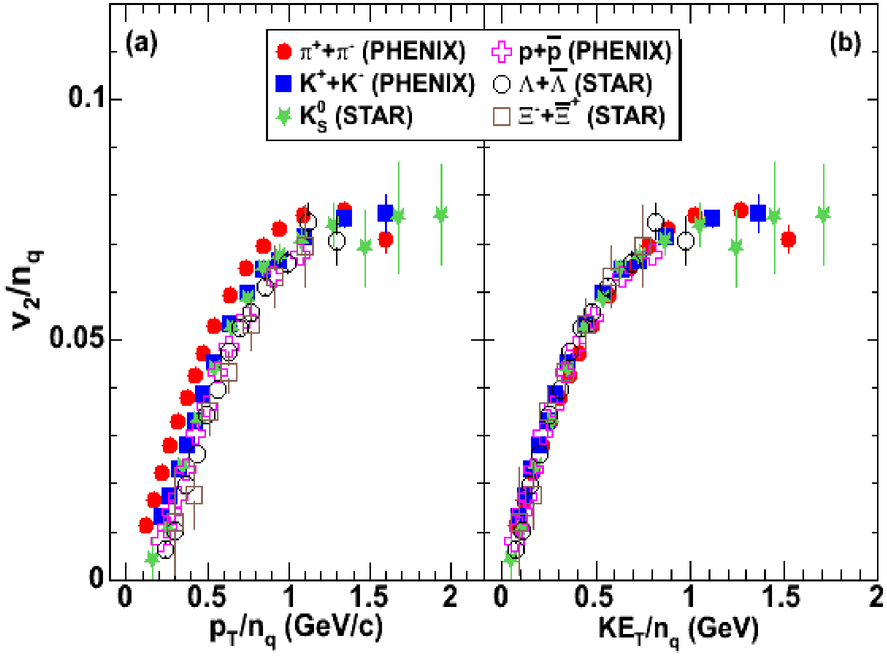
Baryons scale together

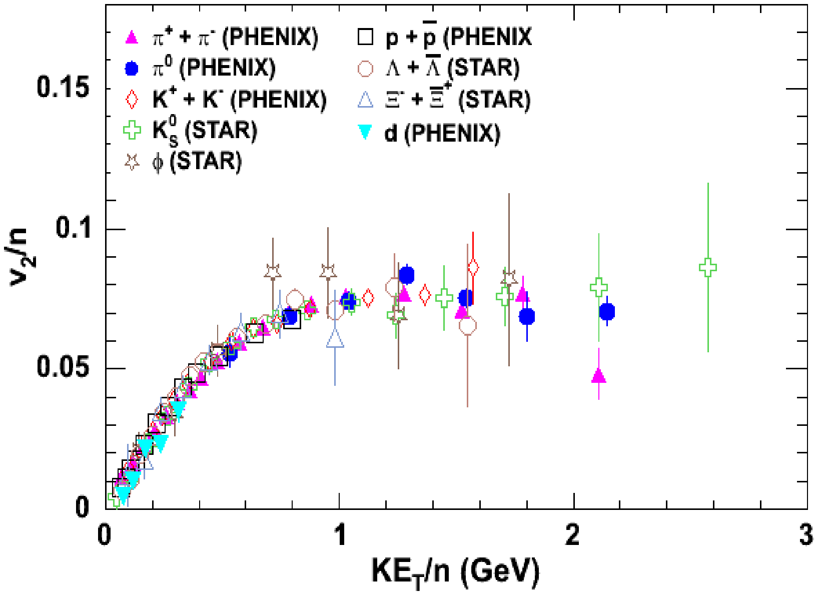
Mesons scale together

Scaling holds up to ~1 GeV

- KE_T scaling is can be viewed as hydrodynamic scaling
- Matter behaves hydrodynamically for $KE_T \leq 1$ GeV
- Hint of partonic degrees of freedom at higher KE_T

Test for partonic degrees of freedom



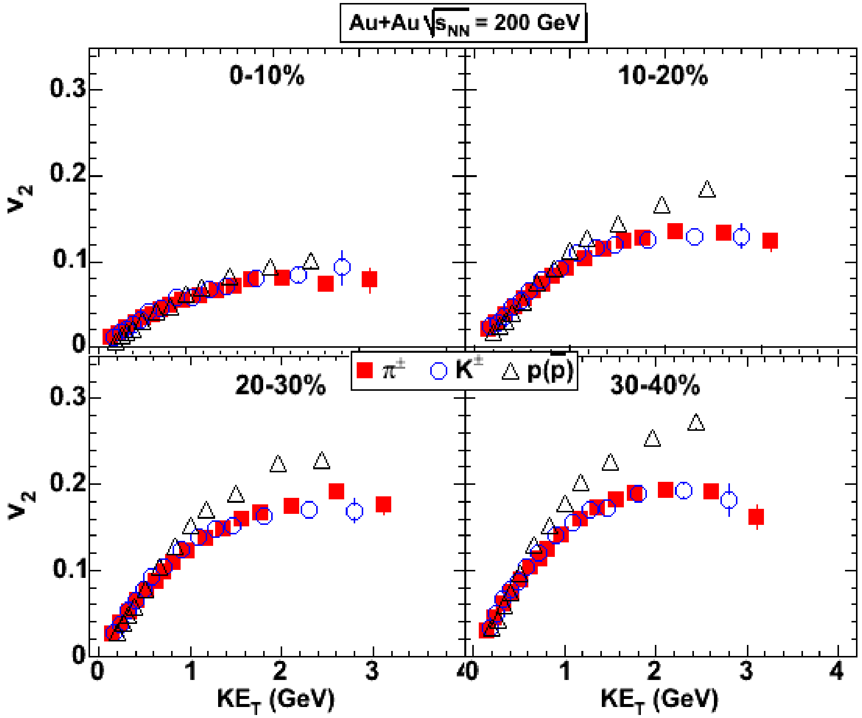


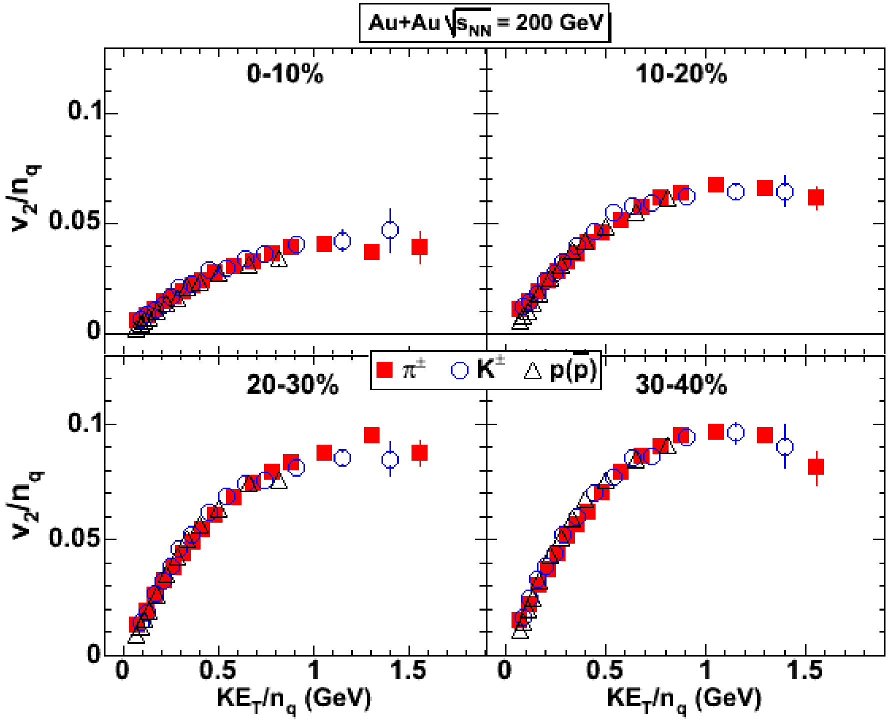
$$v_2^{\,h}\left(KE_T\right) = n v_2^{\,p}\left(KE_T / n\right)$$

KE_T/n gives kinetic energy per quark, assuming that each quark carries equal fraction of kinetic energy of hadron

Scaling holds over the whole range of KE_T and is comprehensive

Kinetic energy scaling: centrality dependence





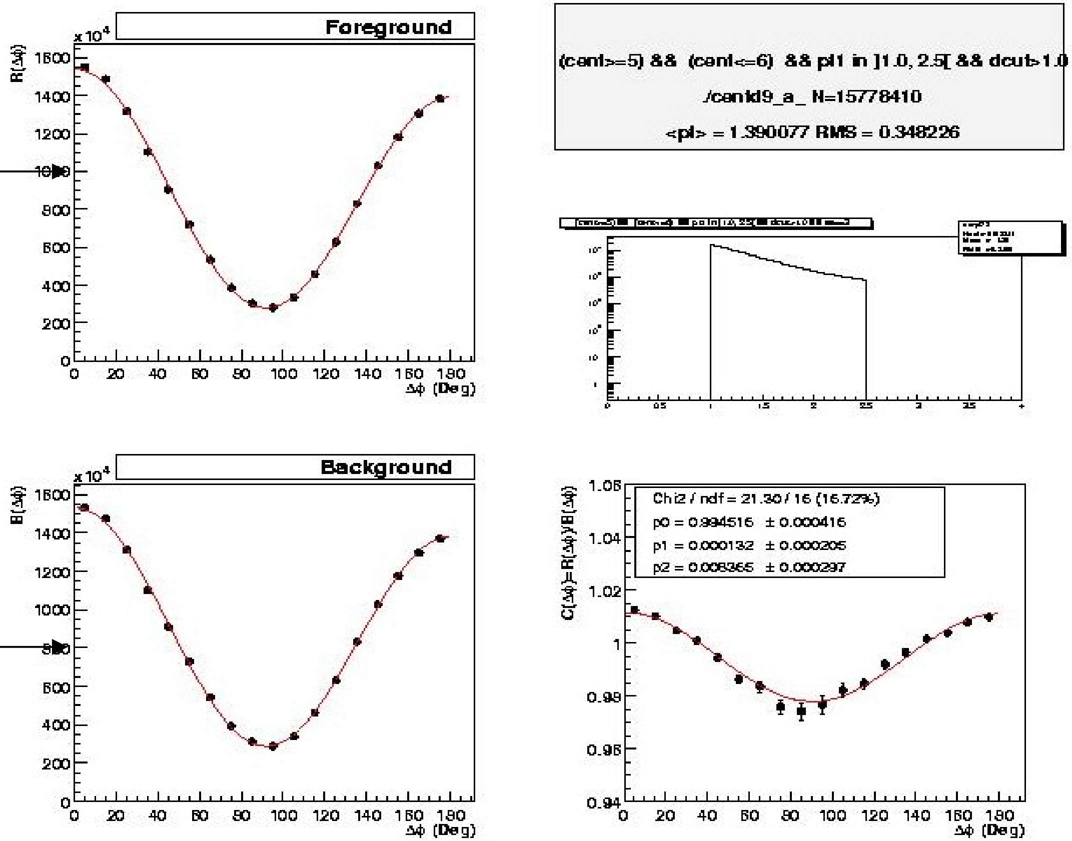
- KE_T scaling breaks at lower KE_T for more peripheral collisions
- KE_T/n scaling holds across the whole KE_T range for centralities presented
- KE_T scaling provides a link between hydrodynamic and recombination mechanisms in the development of flow

Methods to measure elliptic flow

Two-particle correlation method in PHENIX

Au+Au √s=130 GeV

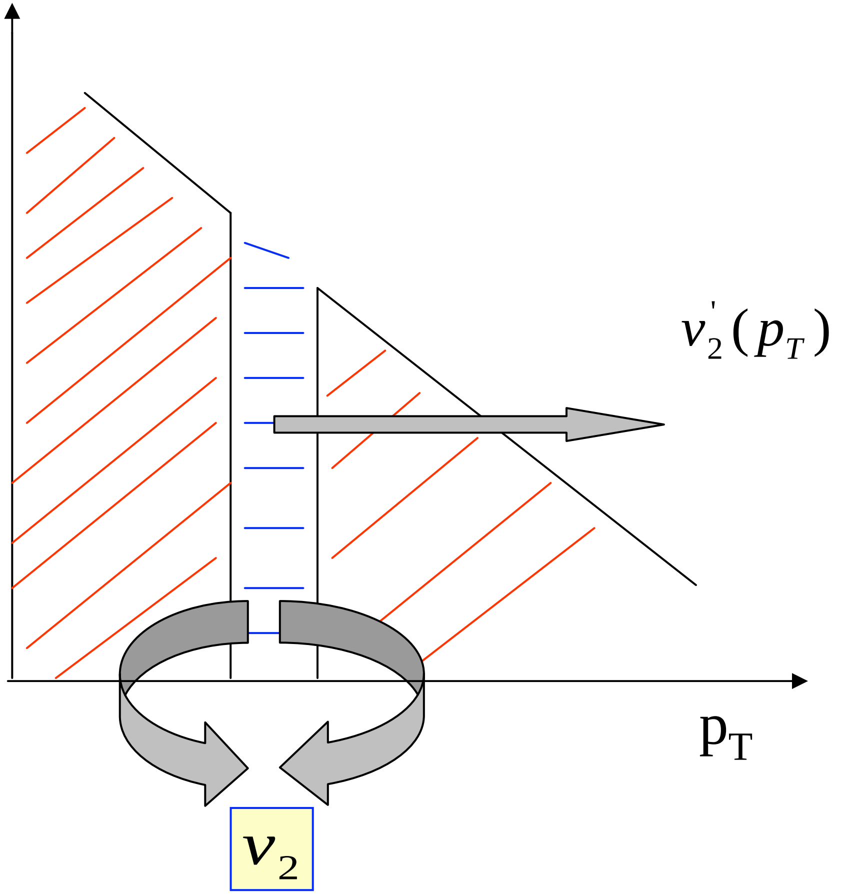
$$C(\Delta\phi) = \frac{N_{real}(\Delta\phi)}{N_{mixed\_events}(\Delta\phi)}$$



Correlation function is fitted with a functional a(1+2v_2 cos(2ΔΦ)), from which v_2 is extracted, a is a normalization constant

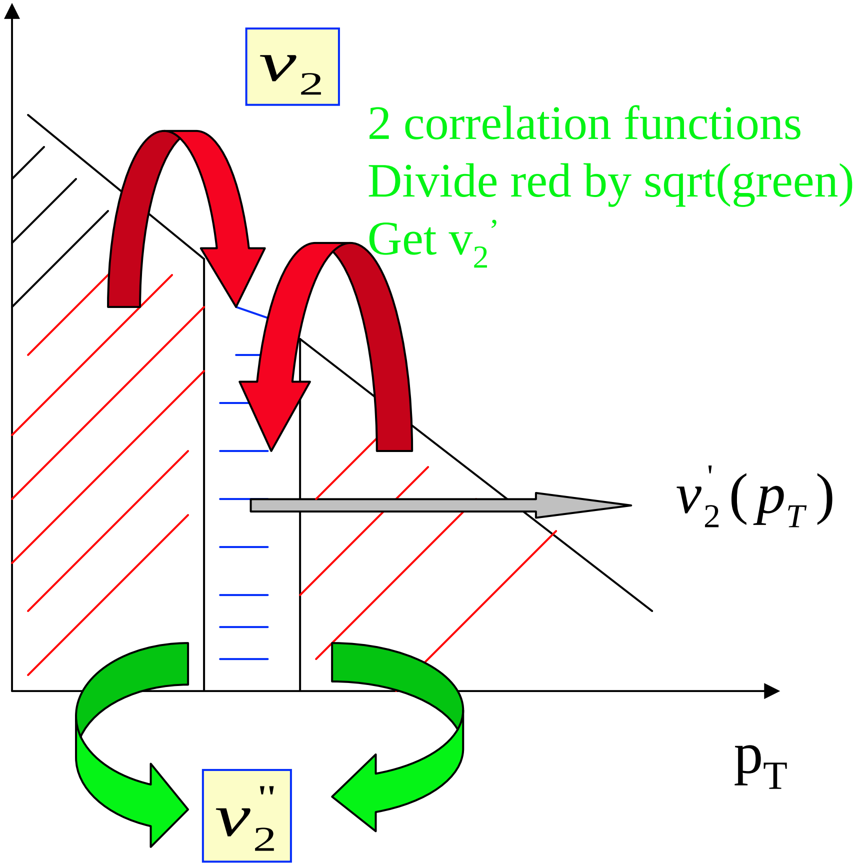
Two-particle correlation methods

Fixed p_T method



$$v_2 = (v_2'(p_T))^2$$

Assorted p_T method



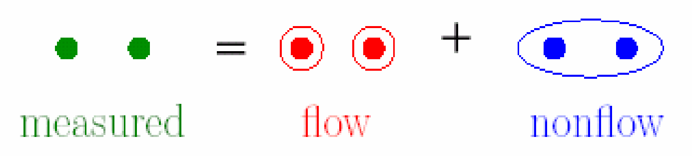
2 correlation functions
Divide red by sqrt(green)
Get v_2'

$$v_2 = v_2'(p_T) \times v_2''$$

Cumulant Method

cumulant method
Borghini, Dinh and Ollitrault
(Phys.Rev.C 64 054901 (2001))
allows for detailed integral and differential measurements of v_2. In this method, flow harmonics are calculated via the cumulants of multiparticle azimuthal correlations and non-flow contributions are removed by higher order cumulants.

Two-particle correlations can be decomposed into a term containing correlations with the reaction plane (flow) and a term corresponding to direct correlations between the particles (non-flow):



$$\left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle_m = v_n^2 + \left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle_c$$

The second order cumulant is defined as:

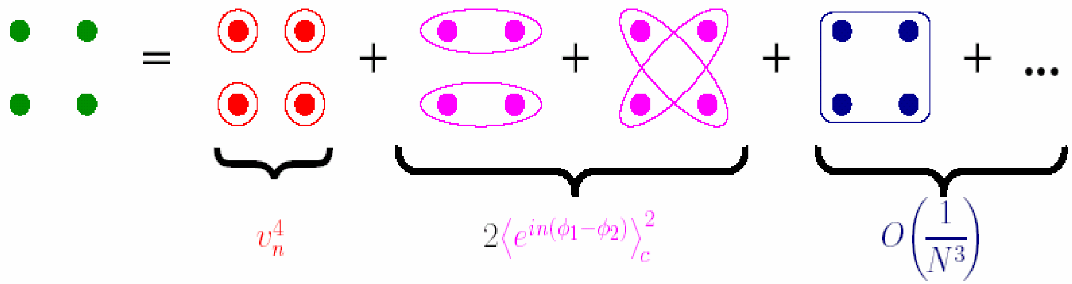
$$c_n\{2\} = \langle\langle e^{in(\phi_1-\phi_2)} \rangle\rangle = v_n^2 + \langle e^{in(\phi_1-\phi_2)} \rangle_c$$

The second term is due to direct correlations between two particles, which may be due to quantum correlations, momentum conservation, jets, etc.

Azimuthal anisotropy from multi-particle correlations

If flow predominates, cumulants of higher order can be used to reduce non-flow contributions

- Following the decomposition strategy presented earlier for two-particle correlations, the 4 particle correlations can be similarly decomposed as follows:



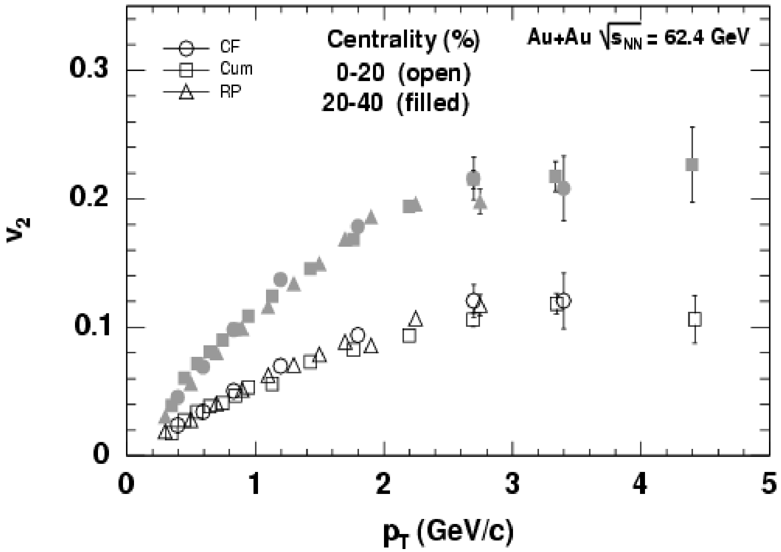
$$C_n\{4\} = \left\langle e^{in(\varphi_1+\varphi_2-\varphi_3-\varphi_4)}\right\rangle - \left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle\left\langle e^{in(\varphi_3-\varphi_4)}\right\rangle - \left\langle e^{in(\varphi_1-\varphi_4)}\right\rangle\left\langle e^{in(\varphi_3-\varphi_2)}\right\rangle$$

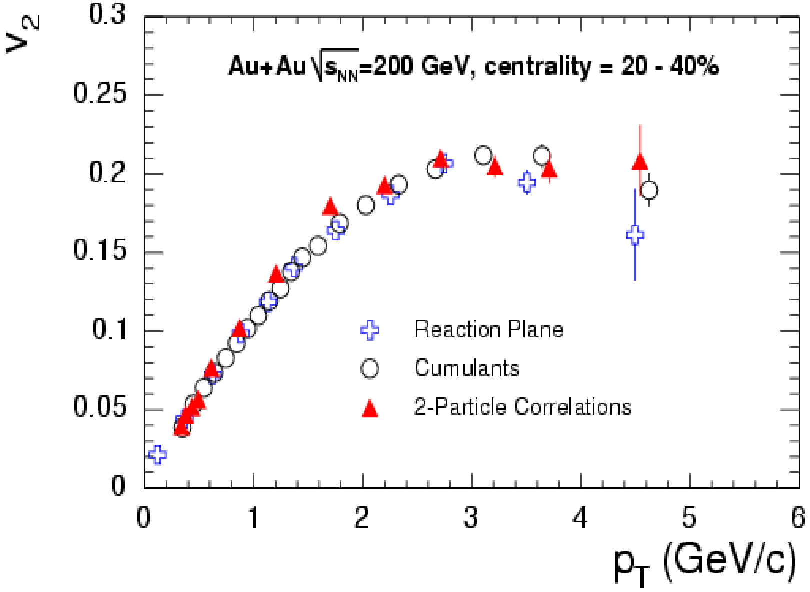
$$\langle\langle e^{2i(\phi_1-\phi_2+\phi_3-\phi_4)}\rangle\rangle \equiv \langle e^{2i(\phi_1-\phi_2+\phi_3-\phi_4)}\rangle - 2\langle e^{2i(\phi_1-\phi_2)}\rangle^2$$

$$= -v_2^4 + O\left(\frac{1}{M^3}\right)$$

Two-particle non-flow contributions removed

Comparison of v_2 obtained from different methods





- Three different methods applied in PHENIX
- RP and cumulant method applied in STAR
- They agree within errors for Au+Au collisions for low p_T

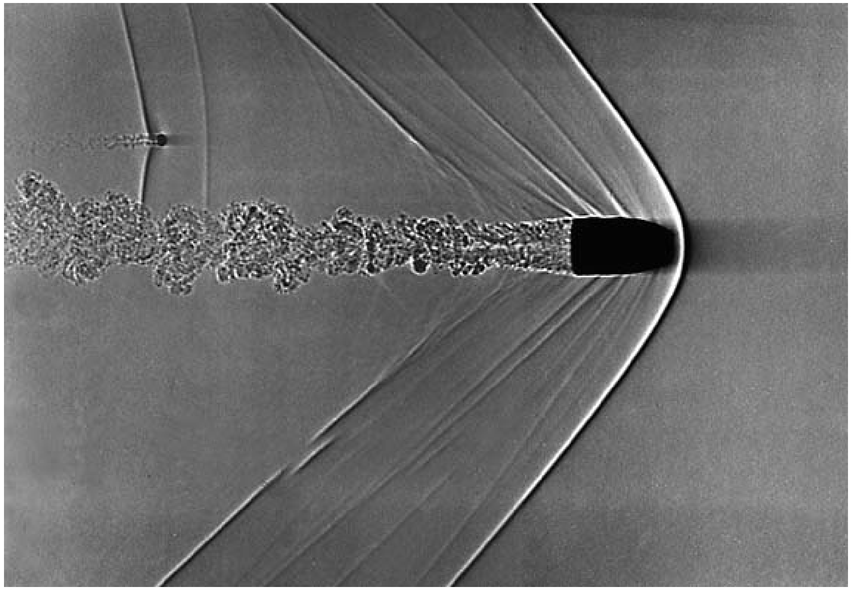
Do we have other handles on c_s ?

What happens to a fast parton moving through the medium?

one idea is that it might generate a shock wave and emit radiation at a characteristic angle that depends on c_s (the speed of sound in the medium) ...

or, that there would be Cerenkov radiation of gluons ...

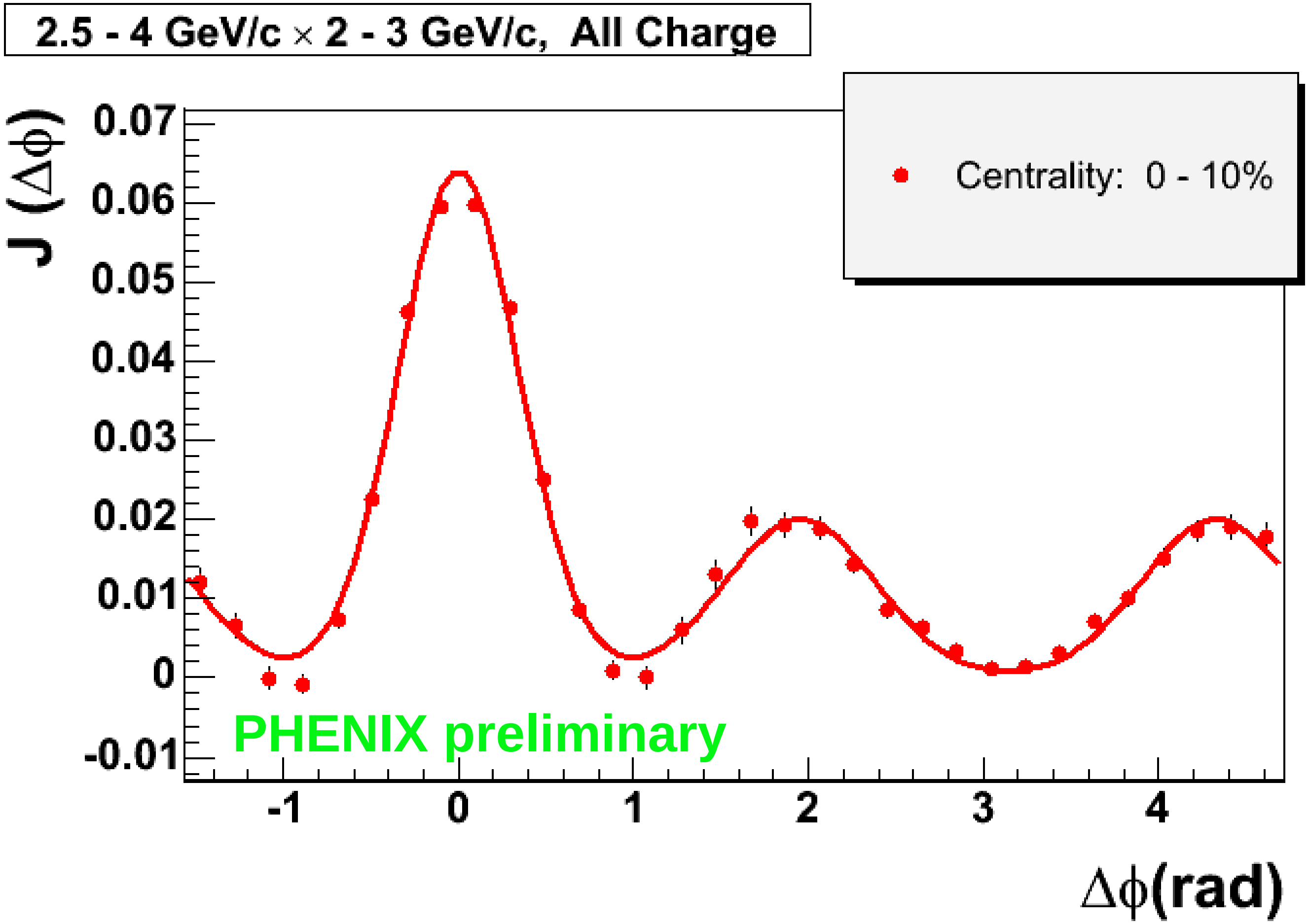
or, that it is deflected in the dense, flowing medium



Casalderrey-Solana, Shuryak and Teaney, hep-ph/0411315

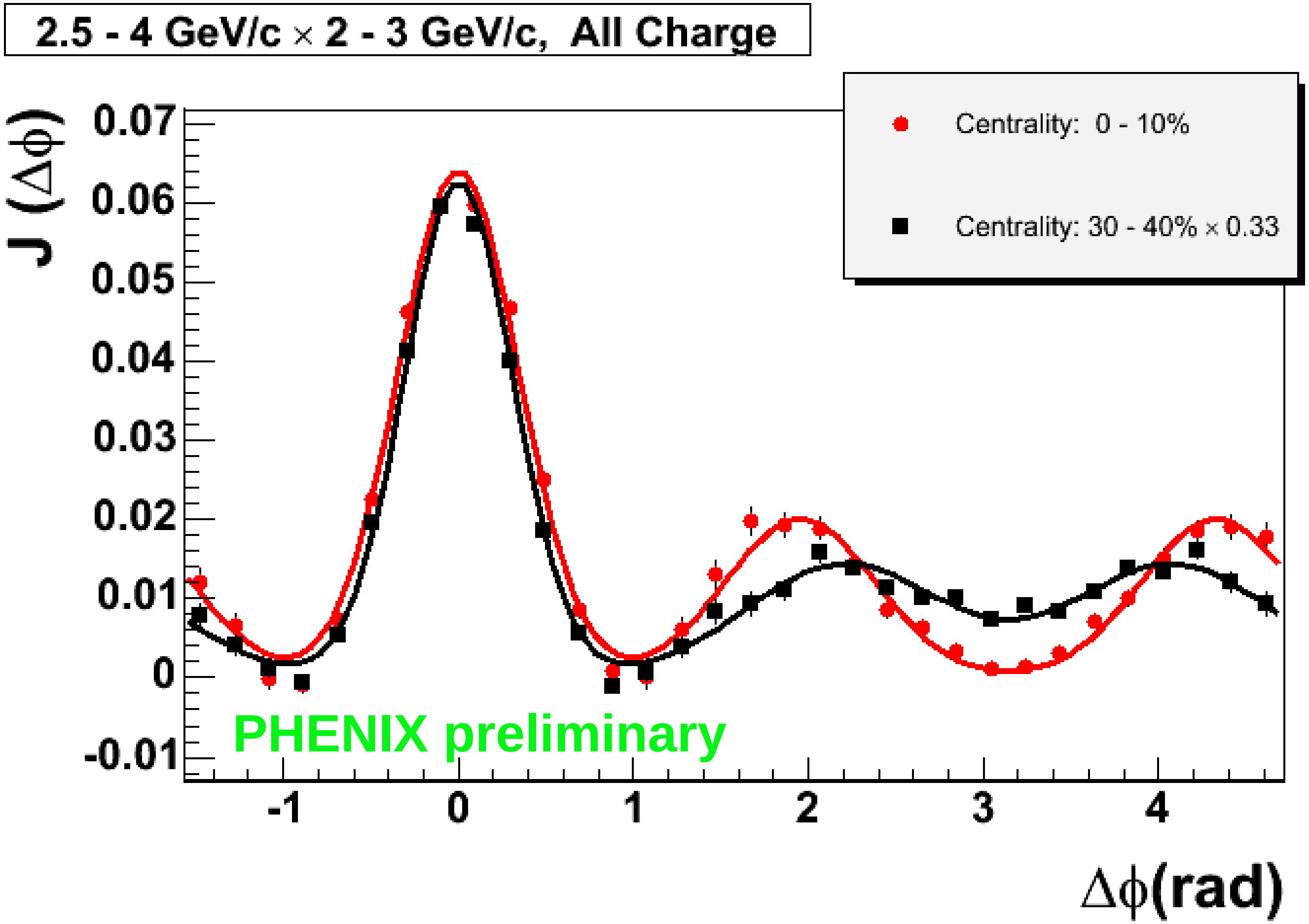
Koch, Majumder, X.-N. Wang, nucl-th/0507063

Jet shape vs centrality



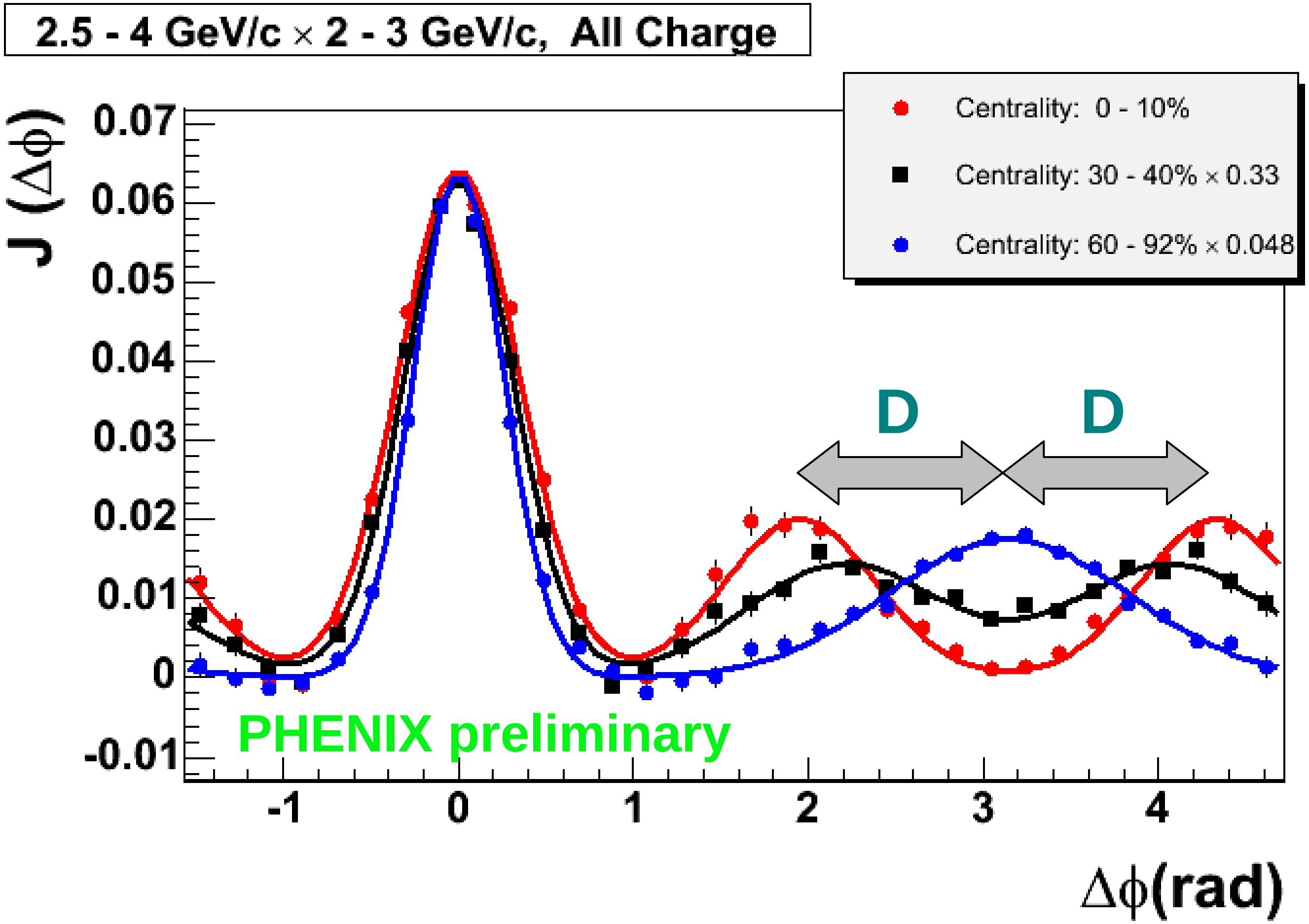
J. Jia

Jet shape vs centrality

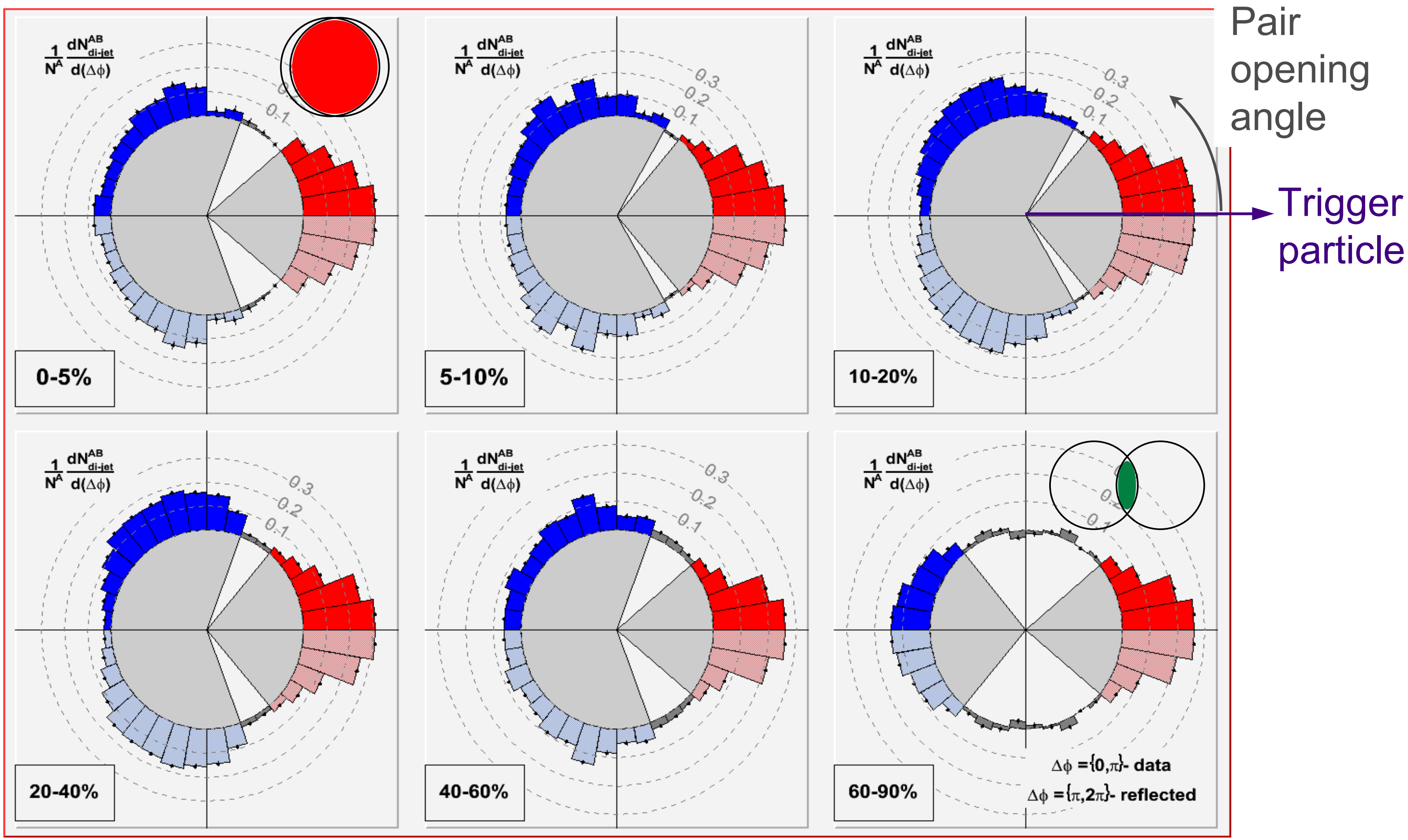


J. Jia

Jet shape vs centrality

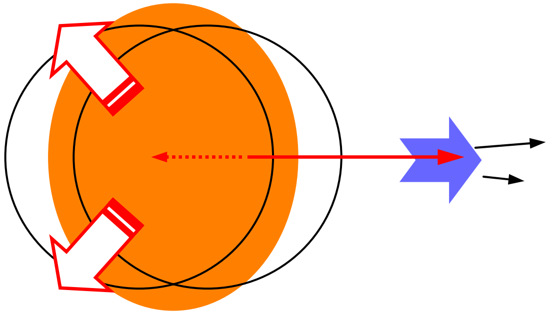


J. Jia Near side : broadening, Away side: splitting



Pair opening angle

Trigger particle



Suggestive of…

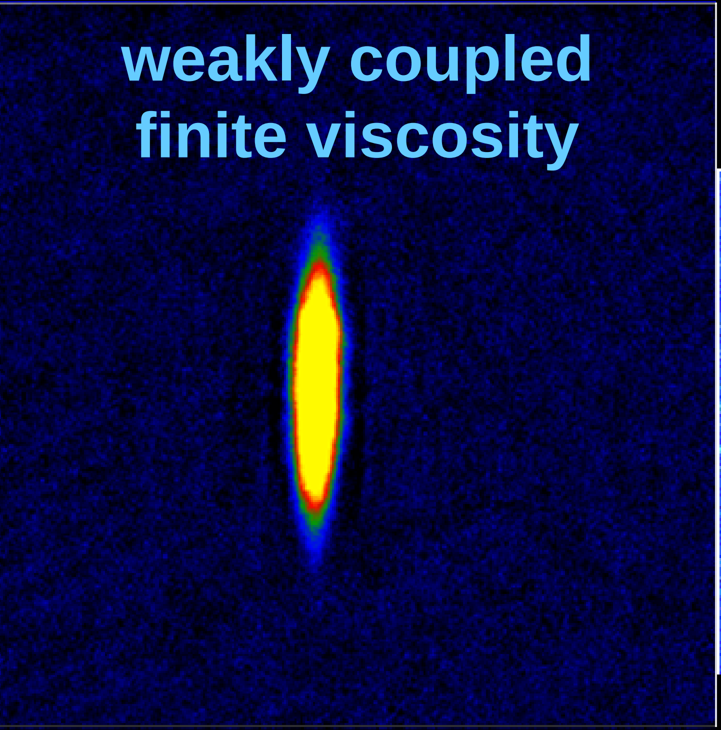
Cherenkov cones?
Mach cones?

The medium ("fluid") appears to have low viscosity

From R. Seto

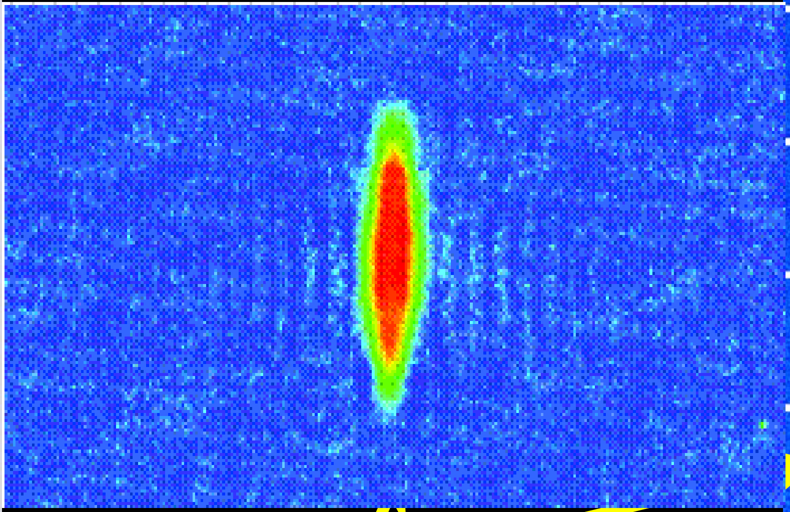
- Same phenomena observed in gases of strongly interacting atoms (Li6)

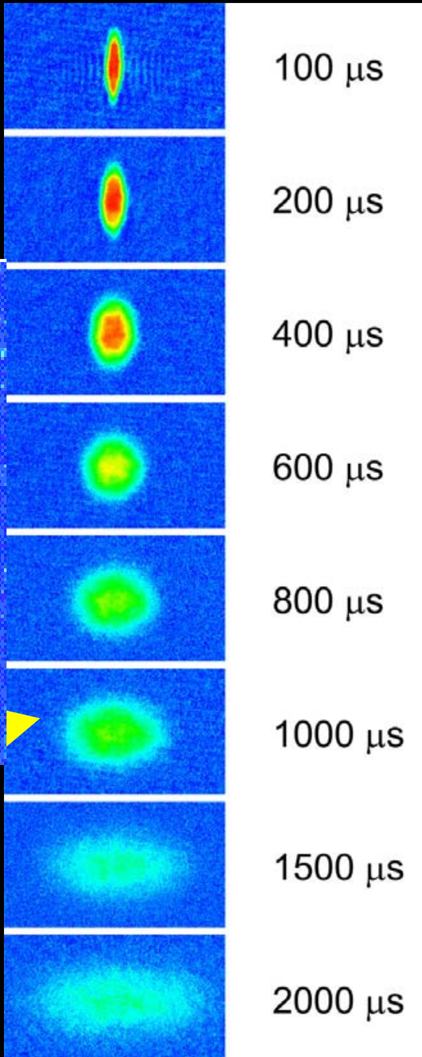
M. Gehm, et al
Science 298 2179 (2002)



weakly coupled
finite viscosity

strongly coupled
viscosity=0





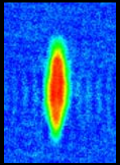
The RHIC fluid behaves like this,

that is, viscocity~0

State of Matter appears strongly interacting

(Similar to a “fluid”)

Once again, in Pictures, what we see in experiment…



→ **Initial spatial anisotropy converted into momentum anisotropy (think of pressure gradients…)**

→ **Efficiency of conversion depends on the properties of the medium**

→ **In particular, the conversion efficiency depends on viscosity**

Pictures from: M. Gehm, et al., Science 298 2179 (2002)